Note Title

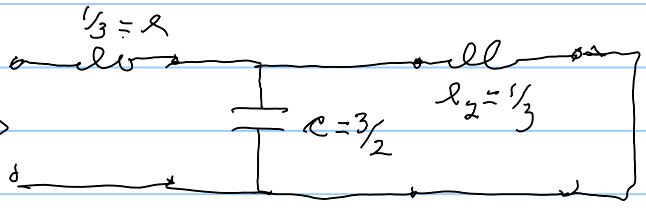
EE610
10/06/16

10/6/2016

$$y(s) = \frac{3(s^2+2)}{s(s^2+4)} = \cfrac{1}{\frac{1}{3}s + \cfrac{1}{\frac{3}{2}s + \cfrac{1}{\frac{1}{3}s + 0}}}$$

Continued fraction expansion about $s = \infty$

$y(s) \rightarrow$
$= 1/z(s)$



1st Cauer

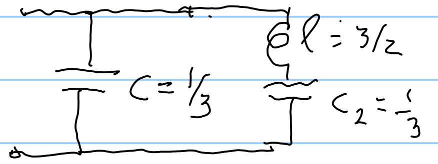
dual $v \Leftrightarrow i$ $z \Leftrightarrow y$

$\Downarrow$

$y^D(s) = z(s)$ example: $z = y^D = \frac{3(s^2+2)}{s(s^2+4}$ $= \cfrac{1}{\frac{1}{3}s + \cfrac{1}{\frac{3}{2}s + \cfrac{1}{\frac{1}{3}s}}}$

$\leftarrow z$
$\leftarrow y$
$\leftarrow z$
$\leftarrow y$

$y = \frac{s(s^2+4)}{3(s^2+2)} \longrightarrow$



dual of the above

in terms of $S(s)$; $2v^i = v + i$, $2v^r = v - i$

$2v^i = 2v^{iD} = i + v$, $2v^{rD} = i - v = -(v - i) = -2v^r$

$v^r = S v^i$ $\quad v^{rD} = S^D v^{iD} = S^D v^i = -v^r$, $v^r = -S^D v^i$

$\Rightarrow S^D = -S$

$S = (1_n + Y)^{-1}(1_n - Y)$, $S^D = (1_n + Y^D)^{-1}(1_n - Y^D)$

$= (1_n + Z)^{-1}(1_n - Z)$

$= (1_n + Y^{-1})^{-1}(1_n - Y^{-1})$

$= (Y^{-1}(Y + 1_n))^{-1} Y^{-1}(Y - 1_n)$

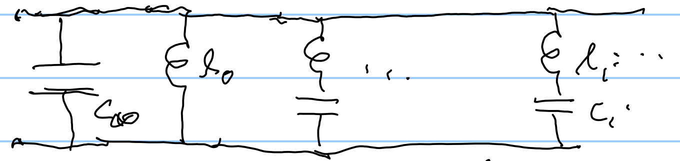
$-S = -(1_n + Y)^{-1}(1_n - Y) = (Y + 1_n)^{-1}(Y - 1_n) = (Y + 1_n)(Y^{-1})^{-1} Y^{-1}(Y - 1_n)$

RC circuits :

convert LC to RC by $L \Rightarrow R$

for LC $y(s) = k_\infty s + \frac{k_0}{s} + \sum \frac{k_i s}{s^2 + \omega_i^2}$

$y_{LC}(s) \longrightarrow$



$l_i = 1/k_i$, $C_i = k_i/\omega_i^2$

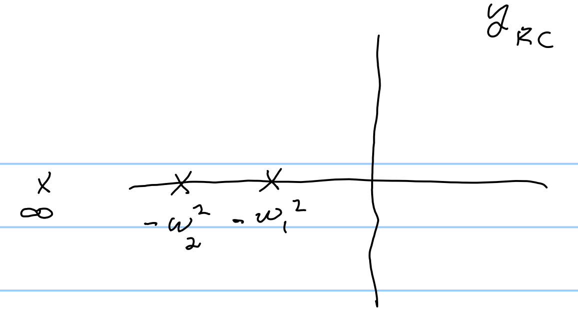
i th one $y_i = \frac{k_i s}{s^2+\omega_i^2} = \frac{1}{\frac{s}{k_i} + \frac{1}{\frac{k_i s}{\omega_i^2}}}$

$y_{RC} \longrightarrow$

$z_i = r_i + \frac{1}{sC_i} = \frac{1}{k_i} + \frac{1}{\frac{k}{\omega_i^2} s}$

$y_i = \frac{k_i s}{s + \omega_i^2}$ $= y_i = \frac{1}{z_i} = \frac{1}{\frac{1}{k_i} + \frac{1}{k_i s/\omega_i^2}} = \frac{k_i s}{\omega_i^2} \Big/ \left[\frac{s}{\omega_i^2} + 1\right]$

$$y_{RC} = k_\infty s + k_0 + \sum \frac{k_i s}{s+\omega_i^2}$$



not in residue ⇒ $\underbrace{\frac{n_i}{s+\omega_i^2}}_{\text{from a partial fraction expansion}}$ not $\underbrace{\frac{k_i s}{s+\omega_i^2}}$ — comes from the circuit form

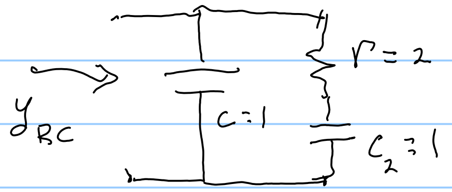
∴ form $\frac{y_{RC}}{s} = k_\infty + \frac{k_0}{s} + \sum \frac{n_i}{s+\omega_i^2}$ this is a partial fraction expansion with simple poles and positive residues

$y_{RC}(s) = \frac{s(s+4)}{s+2}$; $\frac{y_{RC}}{s} = \frac{s+4}{s+2} = 1 + \frac{n_1}{s+2}$

$\times(s+2)\left(\frac{s+4}{s+2}\right) = (s+4) = (s+2)\cdot 1 + n_1$ | set $s=-2$

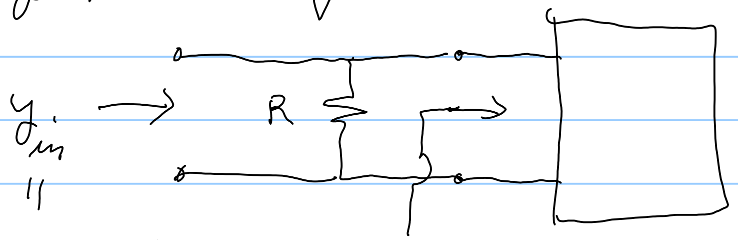
$s=-2 \Rightarrow -2+4 = 2 = 0 + n_1 \Rightarrow n_1 = 2$

$Y_{RC} = s\left(\frac{s+4}{s+2}\right) = s \cdot 1 + \frac{2 \cdot s}{s+2}$



2nd Foster

Hurwitz property



$=$

$G + \frac{sN(s^2)}{D(s)}$ $\quad Y_{LC} = \frac{sN(s^2)}{D(s^2)}$ } assume PR ⇒ Z_{in} no poles in $\sigma > 0$

choose $R = 1 \Rightarrow G = 1$ $\quad Z_{in} = \frac{1}{Y_{in}} = \left.\frac{1}{G + \frac{sN(s^2)}{D(s^2)}}\right|_{G=1} = \frac{D(s^2)}{D(s^2) + sN(s^2)} = \frac{D(s^2)}{P(s)}$

Given a $P(s) = Ev\,P(s) + Od\,P(s) = P_1(s) + s\,P_2(s) \Rightarrow Y = \frac{s\,P_2(s^2)}{P_1(s^2)}$

$P(s) = 3s^5 + 2s^4 + 1s^3 + 3s^2 + 2s + 1$? is this Hurwitz

$= (2s^4 + 3s^2 + 1) + 3s^5 + 1s^3 + 2s$

Form $y(s) = \dfrac{3s^5 + s^3 + 2s}{2s^4 + 3s^2 + 1}$ is this a lossless PR

$$\frac{3}{2}s$$

$$2s^4 + 3s^2 + 1 \overline{\big) \, 3s^5 + s^3 + 2s}$$

$$\underline{3s^5 + \frac{9}{2}s^3 + \frac{3}{2}s}$$

$$\frac{2-9}{2}s^3 + \frac{4-3}{2}s \qquad -\frac{4}{7}s$$

$$\shortparallel$$

$$-7/2\, s^3 + \frac{1}{2}s \overline{\big) \, 2s^4 + 3s^2 + 1}$$

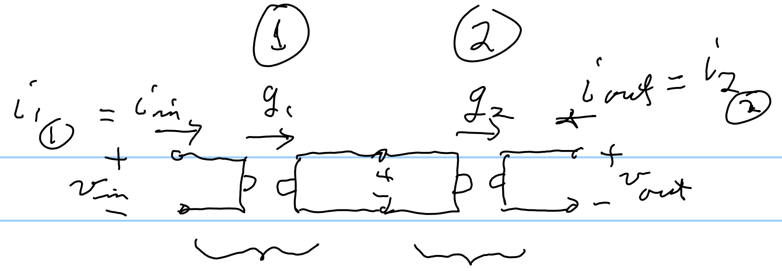
$$\underline{2s^4 - \frac{1}{14}s^2}$$

← gives a negative L in a 1st Cauer synthesis

∴ P(s) is not Hurwitz

i.e. P(s) has a zero in the RHP

cascade of 2 gyrators



$i_{2①} = -i_{1②}$

$= g_1 v_{in} = -(-g_2) v_{out} \Rightarrow v_{out} = \frac{g_1 v_{in}}{g_2}$

$Y_1 = \begin{bmatrix} 0 & -g_1 \\ g_1 & 0 \end{bmatrix}$ $Y_2 = \begin{bmatrix} 0 & -g_2 \\ g_2 & 0 \end{bmatrix}$

$\uparrow$

$i_{1①} = -g_1 v_{2①}$, $v_{2①} = v_{1②}$

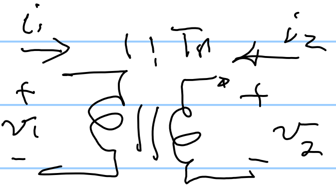
$= -g_1 v_{1②}$

$\frac{g_1}{g_2} = T_n$

$i_{2②} = g_2 v_{1②} \longrightarrow = -g_1 \cdot \frac{i_{2②}}{g_2} = -\frac{g_1}{g_2} \cdot i_{2②}$

$i_{in} = -\frac{g_1}{g_2} i_{out} ; = -T_n \cdot i_{out}$

for a ideal transformer



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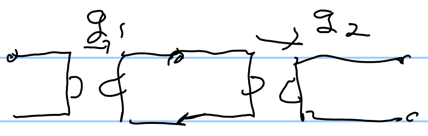

$T_n = g_1 / g_2$

$v_2 = T_n v_1$

$P_{in} = 0 = v_2 i_2 + v_1 i_1$

$= v_1 (T_n i_2 + i_1)$

$i_1 = -T_n i_2$