EE610

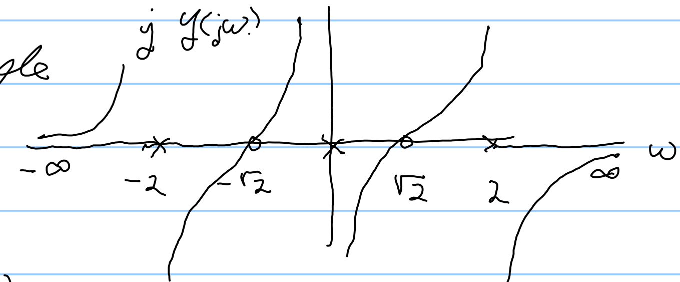
10/04/16

Note Title 10/4/2016

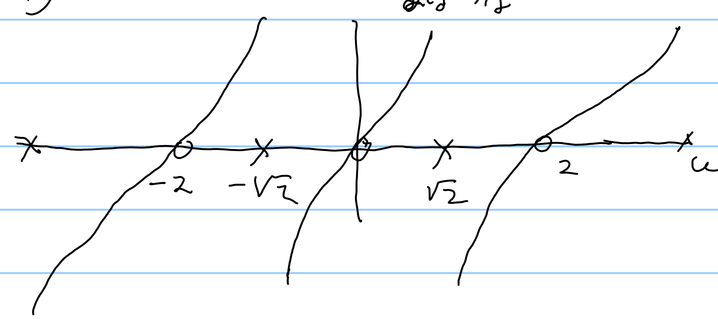
synthesis example

$$y(s) = \frac{3(s^2+2)}{s(s^2+4)}$$

$$jy(j\omega) = \frac{j3}{j\omega}\left(\frac{-\omega^2+2}{-\omega^2+4}\right) = \frac{3(-\omega^2+2)}{\omega(-\omega^2+4)}$$



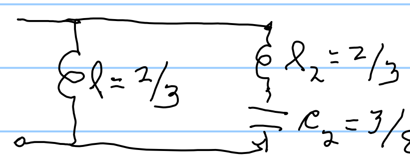
$$z(s) = \frac{1}{y(s)} = \frac{\frac{1}{3}s(s^2+4)}{(s^2+2)}$$



$y(s) \rightarrow$ partial fraction expansion

$$y(s) = \frac{3/2}{s} + \frac{3/4}{s+j2} + \frac{3/4}{s-j2}$$

$$= \frac{3/2}{s} + \underbrace{\frac{3/2\, s}{s^2+4}}_{y_1} \Rightarrow$$



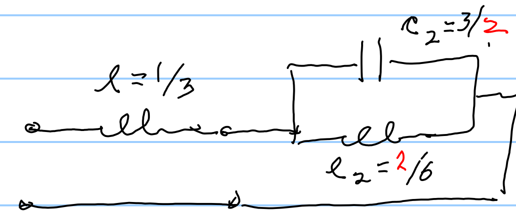
2nd Foster

$$z_1 = \frac{1}{y_1} = \frac{s^2+4}{(3/2)s} = \frac{s}{3/2} + \frac{1}{\frac{3}{8}s}$$

$$z(s) = \frac{1}{y(s)} = \frac{\frac{1}{3}s(s^2+4)}{s^2+2} = \frac{1}{3}s + \frac{2k_1 s}{s^2+2} \Rightarrow \left(\cancel{\frac{s^2+2}{s}}\right)\left(\frac{\frac{1}{3}\cancel{s}(s^2+4)}{\cancel{s^2+2}}\right)\Bigg|_{s^2=-2} = \frac{1}{3}s\left(\frac{s^2+2}{s}\right)\Bigg|_{s^2=-2} + 2k_1$$

$$\frac{s^2+4}{3}\Bigg|_{s^2=-2} = 0 + 2k_1 \Rightarrow 2k_1 = 2/3$$

$$z(s) = \frac{1}{3}s + \frac{2/3\, s}{s^2+2} \Rightarrow$$



1st Foster uses minimum # of L's & C's

$$y_1 = 1/z_1 = \frac{s^2+2}{2/3\, s} = \frac{3}{2}s + \frac{1}{\frac{2}{6}s}$$

1st Cauer ⇒ remove poles @ ∞

$$\frac{1}{3}s \quad \leftarrow z$$

$$s^2+2 \;\Big)\; \frac{1}{3}s^3 + \frac{4}{3}s$$

$$\frac{1}{3}s^3 + \frac{2}{3}s$$

$$\frac{2}{3}s$$

$$\frac{3}{2}s \quad \leftarrow y$$

$$\frac{2}{3}s \;\Big)\; s^2+2$$

$$s^2$$

$$2$$

$$\frac{1}{3}s \quad \leftarrow z$$

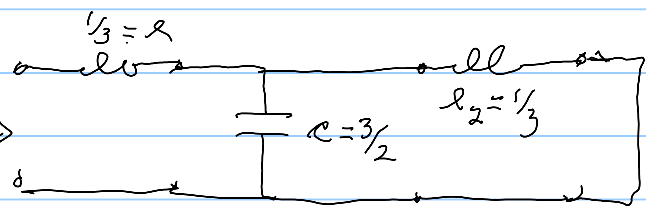
$$2 \;\Big)\; \frac{2}{3}s$$

$$\frac{2}{3}s$$

$$0 \quad \leftarrow y$$

$$y(s) = \frac{3(s^2+2)}{s(s^2+4)} = \cfrac{1}{\frac{1}{3}s + \cfrac{1}{\frac{3}{2}s + \cfrac{1}{\frac{1}{3}s + 0}}}$$

Continued fraction expansion about $s = \infty$



$y(s) \rightarrow$

$= 1/z(s)$

1st Cauer

2nd Cauer $\Rightarrow$ continued fraction expansion about $s = 0$

$$y(s) = \frac{3(s^2+2)}{s(s^2+4)} = \frac{3s^2+6}{s^3+4s}$$

$$\frac{6}{4s} \leftarrow y$$

$$4s + s^3 \;\big)\; 6 + 3s^2$$

$$6 + \frac{6}{4}s^2$$

$$\frac{8}{3s} \leftarrow z$$

$$\frac{3}{2}s^2 \;\big)\; 4s + s^3$$

$$4s$$

$$\frac{3}{2s} \leftarrow y$$

$$s^3 \;\big)\; \frac{3}{2}s^2$$

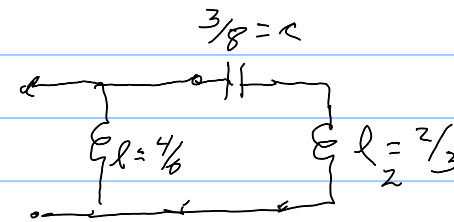
$$+0$$

$$y(s) = \frac{6}{4s} + \cfrac{1}{\cfrac{8}{3s} + \cfrac{1}{\cfrac{3}{2s} + 0}}$$

$\leftarrow y$

$\leftarrow z$

$\leftarrow y$



2nd Cauer

circulator: $S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

3-port

lossless $\Rightarrow S(\alpha) S^T(-\alpha) = 1_3 \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

find Y or Z

$$Y = (1_3 + S)^{-1} (1_3 - S)$$

$1_3 + S = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$, $1_3 - S = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$\det(1_3 + S) = 1 \cdot \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 1 - (-1) = 2$

the $(2,1)$ entry of $(1_3 + S)$ we need the $(1,2)$ cofactor $= (-1)^{1+2} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$

for all: $(1_3 + S)^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

$$Y = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & +2 & -2 \\ -2 & 0 & 2 \\ +2 & -2 & 0 \end{bmatrix} = -Y^T$$ is skew symmetric

lossless condition is $Y(s) + Y^T(-s) = 0 \Rightarrow Y = -Y^T$ here

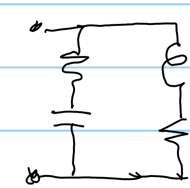
$\Rightarrow$ 3-port circulator is lossless

$$Y = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ +1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \Leftarrow$$ 3 gyrators

$\therefore$ look @ degree 1 PR $y(s) = \dfrac{as+b}{cs+d}$, $a, b, c, d \geq 0$

$$= \frac{as}{cs+d} + \frac{b}{cs+d} \quad \text{each is PR}$$

$$= \frac{1}{\frac{c}{a} + \frac{d}{as}} + \frac{1}{\frac{c}{b}s + d/b}$$



$$T(s) = \frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]} = \frac{Y(s)}{U(s)} = \frac{N(s)}{D(s)} = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}$$

$$= \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]}$$

$\Rightarrow$ differential eq. $\left[b_n \frac{d^n y}{dt^n} + \dots + b_0 y\right] = a_m \frac{d^m u}{dt^m} + \dots + a_1 \frac{du}{dt} + a_0 u$

bilateral $\mathcal{L}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-st}\, dt$ if $f(t) = 0$ for $t < 0$

$\Rightarrow f_1(t) 1(t)$, $1(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

unilateral $\mathcal{L}[f_1(t) \underline{1(t)}] = \int_0^{\infty} f_1(t) e^{-st}\, dt$