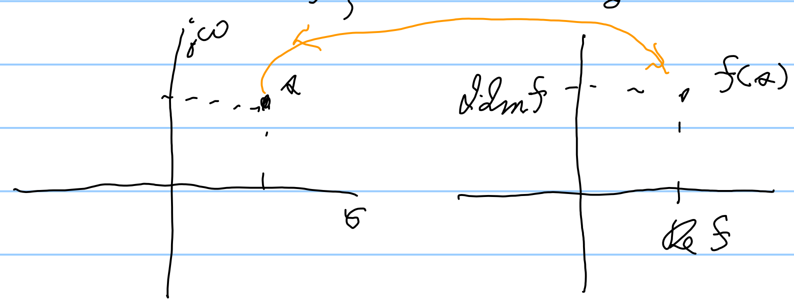
EE 610
09/29/16

Note Title 9/29/2016

PR $f(s)$, $s = \sigma + j\omega$ $\quad f(s) = \mathrm{Re}\, f(s) + j\, \mathrm{Im}\, f(s)$

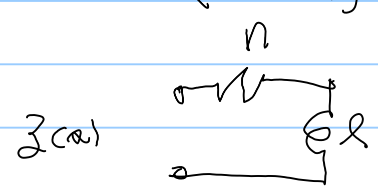


If $f(s)$ is PR then
$\mathrm{Re}\, f(s) \geq 0$ in $\sigma > 0$

Ex: $z(s) = r + s\ell \Rightarrow y(s) = \dfrac{1}{z(s)} = \dfrac{1}{r + s\ell}$

$$z(s) = r + \sigma\ell + j\omega\ell$$
$$= (r + \sigma\ell) + j\omega\ell$$

$\Big\}$ $\mathrm{Re}\, z(s) = r + \sigma\ell$ in $\sigma > 0$
is PR if r & $\ell \geq 0$

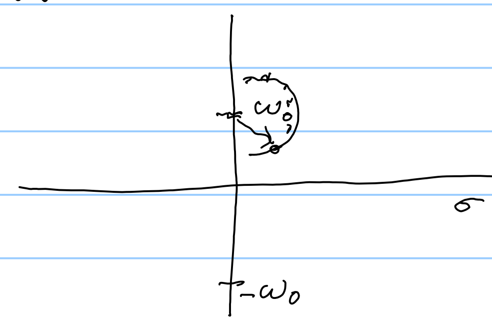


$$y(s) = \frac{1}{r + (\sigma + j\omega)\ell} = \frac{1}{(r + \sigma\ell) + j\omega\ell} \times \frac{(r + \sigma\ell) - j\omega\ell}{(r + \sigma\ell) - j\omega\ell}$$

$$= \frac{r + \sigma\ell}{(r + \sigma\ell)^2 + \omega^2\ell^2} + j\frac{(-\omega\ell)}{(r + \sigma\ell)^2 + \omega^2\ell^2}$$

to check $y(s)$ for PR $\quad \mathrm{Re}\, y(s) = \dfrac{r+\sigma l}{(r+\sigma l)^2 + (\omega l)^2} \geq 0$ in $\sigma \geq 0$ if $r, l \geq 0$

the inverse, $1/g(s)$, is PR if $g(s)$ is PR

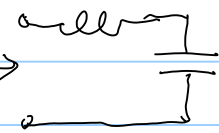


$$y(s) = \frac{s}{s^2+\omega_0^2} = \frac{k_1}{s+j\omega_0} + \frac{k_2}{s-j\omega_0}$$

$$\Rightarrow k_1 = 1/2 = k_2$$

$$= \frac{1/2}{s+j\omega_0} + \frac{1/2}{s-j\omega_0}$$

$$= \frac{1}{z(s)} = \frac{1}{s + \omega_0^2/s} \Rightarrow z(s) = s + \frac{\omega_0^2}{s}$$

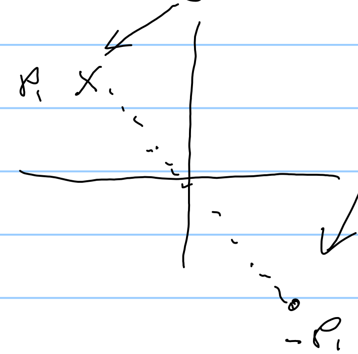
$z(s) \longrightarrow$ 

for a PR then poles on $j\omega$ axis are simple with a positive & real residue. also true for zeros being simple if on

$s = j\omega$.

If lossless all poles are on the $j\omega$ axis because

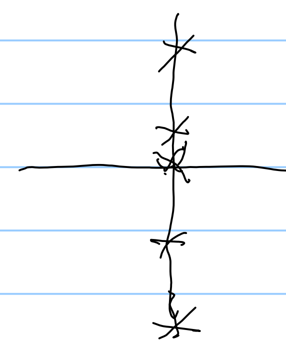
$$Y(s) + Y(-s) = 0 \qquad Y(s) = -Y(-s)$$



would have a pole here but these are poles of $Y(s)$ & can't be if PR

$\Rightarrow$ poles of a lossless $Y(s)$ are all on $j\omega$ axis as are the zeros.

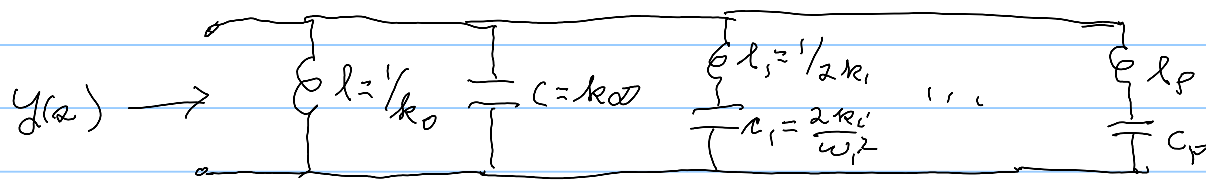
make a partial fraction expansion



$$Y(s) = \sum_{i=1}^{p} \left(\frac{k_i}{s + j\omega_i} + \frac{k_i}{s - j\omega_i} \right) + \frac{k_0}{s} + k_\infty s$$

$$y(s) = \sum_{i=1}^{p} \left(\frac{2k_i s}{s^2 + \omega_i^2} \right) + \frac{k_0}{s} + k_\infty s$$

$$y(s) = \sum_{i=1}^{p} \frac{1}{\frac{s}{2k_i} + \frac{\omega_i^2}{2k_i s}} + \frac{k_0}{s} + k_\infty s$$



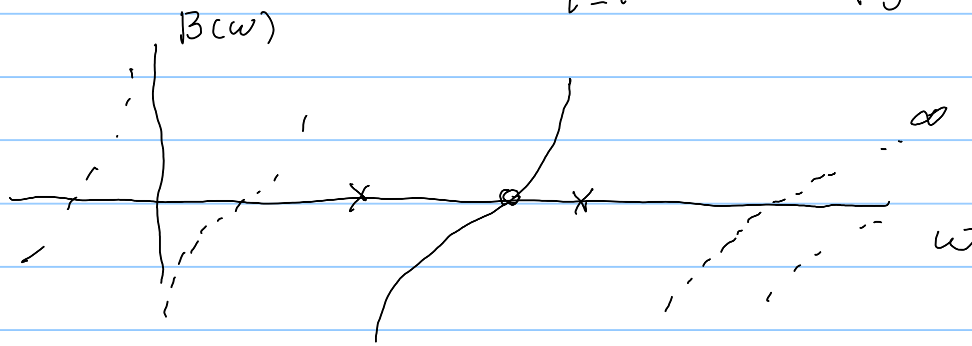
Poles & zeros alternate on $j\omega$ axis: set $s = j\omega$

$$y(j\omega) = \sum_{i=1}^{p} \frac{2k_i j\omega}{-\omega^2 + \omega_i^2} - j\frac{k_0}{\omega} + jk_\infty \omega = jB(\omega)$$

$$\frac{dB(\omega)}{d\omega} = k_\infty + \frac{k_0}{\omega^2} + \sum_{i=1} \left[\frac{2k_i}{(-\omega^2 + \omega_i^2)} - \frac{2k_i\omega(-2\omega)}{(-\omega^2 + \omega_i^2)^2} \right]$$

$$\frac{2k_i(-\omega^2+\omega_i^2)+2k_i\omega^2}{(-\omega^2+\omega_i^2)^2}=\frac{2k_i\omega_i^2}{(-\omega^2+\omega_i^2)^2}>0$$

$$\frac{dB(\omega)}{d\omega}=k_\infty+\frac{k_0}{\omega^2}+\sum_{i=1}^{p}\frac{2k_i\omega_i^2}{(-\omega^2+\omega_i^2)^2}\geq 0 \text{ or } \infty \text{ on } s=j\omega$$



shows a zero between any two poles.

Ex: $Y(s)=\dfrac{3(s^2+2)}{s(s^2+4)}$ to synthesize

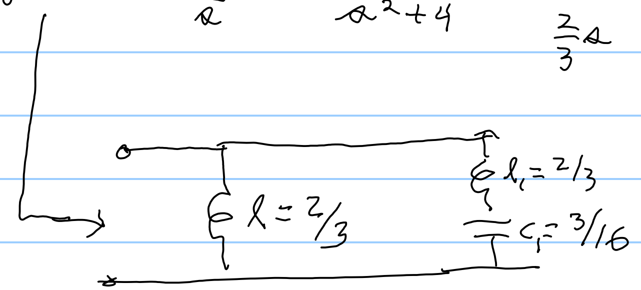
$$=\frac{k_0}{s}+\frac{k_2}{s+j2}+\frac{k_2}{s-j2}=$$

to find $k_0 \Rightarrow \times s \Rightarrow \dfrac{3}{s}\left(\dfrac{s^2+2}{s^2+4}\right)s=k_0+k_2 s\left[\dfrac{1}{s+j2}+\dfrac{1}{s-j2}\right]$

$$3\left(\frac{s^2+2}{s^2+4}\right)\Big|_{s=0} = 3\times\frac{2}{4} = \frac{3}{2} = k_0 + 0 \qquad k_0 = 3/2$$

$$\times\ s+j2 \Rightarrow \quad \frac{3(s^2+2)}{s(s-j2)}\Big|_{s=-j2} = \frac{3(-4+2)}{(-j2)(2\times(-j2))} = \frac{-2\times 3}{-8} = \frac{3}{4} = k_2 + \frac{k_0(0)}{s} + \frac{k_2\cdot 0}{s-j2}$$

$$y(s) = \frac{3/2}{s} + \frac{2\times 3/4\, s}{s^2+4} = \frac{1}{\frac{2}{3}s} + \frac{1}{\frac{2}{3}s + \frac{4}{2s/3}} = \frac{1}{\frac{2}{3}s} + \frac{1}{\frac{2}{3}s + \frac{16}{3s}}$$



= 2nd Foster synthesis of lossless $y(s)$ $\qquad \frac{4}{\frac{3}{4}s} = \frac{16}{3s}$

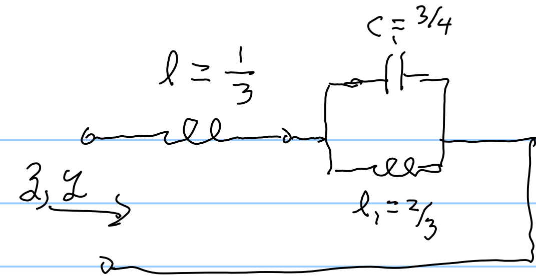
to synthesize from $z(s)$ by partial fraction expansion:

$$z(s) = y(s) = \frac{s(s^2+4)}{3(s^2+2)} = \frac{1}{3}s + \frac{2k_1 s}{s^2+2} = \frac{1}{3}s + \frac{\frac{4}{3}s}{s^2+2} = \frac{1}{3}s + \frac{1}{\frac{3s}{4} + \frac{2}{\frac{4}{3}s}}$$

$$k_i: \frac{s^2+2}{s} \Rightarrow \frac{s(s^2+4)}{3(s^2+2)} \times \frac{s^2+2}{s} = \frac{1}{3}s\times\frac{s^2+2}{s} + 2k_1$$

$$\text{set } s^2 = -2 \Rightarrow \frac{s^2+4}{3} = \frac{1}{3}(s^2+2) + 2k_1 \Rightarrow \frac{-2+4}{3} = 0 + 2k_1,\ k_1 = 2/3$$

$$z(\lambda) = \frac{1}{y(\lambda)} = \frac{1}{3}\lambda + \cfrac{1}{\frac{3}{4}\lambda + \cfrac{1}{\frac{2}{3}\lambda}}$$



1st Foster