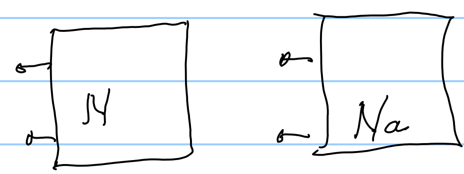
EE610
09/27/16b

Note Title 9/27/2016

adjoint



$$C^T K_a = 0_{b\times b} \Rightarrow Y_a = Y^T \quad ; \quad S = (1_b + Y)^{-1}(1_b - Y)$$

$$C_a^T K = 0_{b\times b}$$

$$S_a = (1_b + Y_a)^{-1}(1_b - Y_a)$$

$$= (1_b + Y^T)^{-1}(1_b - Y^T)$$

$$S^T = (1_b - Y^T)(1_b + Y)^{-T}$$

$$= (1_b + Y^T)^{-1}(1_b - Y^T)$$

$$\Leftrightarrow S_a = S^T$$

$$\mathcal{L}[f(t)] = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt = \int_{-\infty}^{\infty} f(t)e^{-\sigma t - j\omega t}\,dt = \int_{-\infty}^{\infty} [f(t)e^{-\sigma t}]e^{-j\omega t}\,dt$$

$= \mathcal{F}[f(t)e^{-\sigma t}]$ if $\sigma > 0$ $\Rightarrow$ if $f(t) = 0$ for $t < 0$ & square integrable (= finite energy)

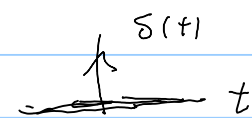
then $f(t)e^{-\sigma t}$ is also finite energy

If a circuit is <u>causal</u> [$v^r(t) = 0$ for $t < 0$ if $v^i(t)$ is 0 for $t < 0$ no response before an excitation]

$\Rightarrow$ $S(t)$ is zero for $t < 0$; physically $S(t)$ is $v^r(t)$ for $v^i(t)$ being an impulse

$$v^r(t) = \int_{-\infty}^{\infty} S(\tau)\,\delta(t-\tau)\,d\tau \quad \text{for} \quad v^i(t) = \delta(t) = \text{unit impulse}$$

$$\Rightarrow S(t) \cdot \int_{-\infty}^{\infty} \delta(t-\tau)\,d\tau$$



If $f(t) = v^i(t)$ is square integrable & 0 for $t < 0$ then for passivity so is $v^r(t)$ & $v^r(t)e^{-\sigma t}$ also if $\sigma > 0$

thus $\|v^i\|^2 - \|v^r\|^2 \geq 0$ if passive, here $\|f\|^2 = \int_{-\infty}^{\infty} f^*(t) f(t)\, dt$

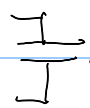
$\left({}^* \Rightarrow j \to -j\right)$ $\quad 1_n - S^{T*}(j\omega) S(j\omega)$ is positive semidefinite

then also if causal $\quad 1_n - S^T(\sigma - j\omega)\, S(\sigma + j\omega)$ for $s = \sigma + j\omega$

$= 1_n - S^{T*}(s) S(s)$ with $\sigma > 0$

for a 1-port, $n = 1$, $S(s)$ is the reflection coefficient

here $S^{T*}(s) = S^T(s^*)$ for $\sigma > 0$ & real circuit

Ex:  C $\quad S(s) = \dfrac{1 - sC}{1 + sC}$ $\quad$ then $S^*(s) = \dfrac{1 - s^* C^*}{1 + s^* C^*} = \dfrac{1 - s^* C}{1 + s^* C} = S(s^*)$

for real C

If $S(s)$ is rational & comes from a passive circuit

then it is called BR (= bounded real & rational)

Bounded Real conditions (BR if rational)

1. $S^*(s) = S(s^*)$ in $\sigma > 0$ (the RHP) [real circuit]
2. $S(s)$ is analytic in $\sigma > 0$ [stable; no poles in $\sigma \geq 0$ if BR]
3. $1_n - S^{T*}(s) S(s)$ is positive semidefinite [passive]

$\Rightarrow$ if $n = 1$, $1 - |S(s)|^2 \geq 0$ in $\sigma > 0$

Known result: If BR then can synthesize by a passive circuit:

If lossless $1_n - S^{T*}(j\omega)S(j\omega) = 0_n$

$$\Rightarrow \quad 1_n - S^T(-s)S(s) = 0_n \quad \Rightarrow \quad S^{-1}(s) = S^T(-s)$$

Eg: c $\Rightarrow$ $S(s) = \frac{1-sc}{1+sc}$, $S(-s) = \frac{1-(-s)c}{1+(-s)c} = \frac{1+sc}{1-sc} = \frac{1}{S(s)}$

Lossless synthesis via $y(s)$ if $S(s)$ is bounded real

$\Rightarrow y(s) + y^{T*}(s) \geq 0$ in $\sigma > 0$

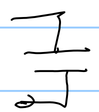
$\equiv$ to $P_{ave}\big|_{s=j\omega} \geq 0$ (average power in sinusoidal steady state)

$\Rightarrow y(s)$ are positive-real if from a passive network (if rational $\Rightarrow$ PR)

PR also means physically realizable.

If lossless $Y(s) + Y^{T*}(s) = O_n$ on $s = j\omega$

$\Rightarrow Y(s) + Y^{T}(-s) = O_n$ in $s = \sigma + j\omega$, $\sigma > 0$

Ex:  C $\quad y(s) = \underbrace{Cs}_{PR} \quad y(-s) = \underbrace{-sC}_{\text{not PR}} \Rightarrow y(s) + y(-s) \equiv 0$

Positive real conditions

1. $Y^{*}(s) = Y(s^{*})$ in $\sigma > 0$ [real circuit]
2. $Y(s)$ is analytic in $\sigma > 0$ [stable circuits]
3. $Y(s) + Y^{*T}(s)$ is positive semidefinite in $\sigma > 0$ [passivity]

If rational $\Rightarrow$ PR, If lossless LPR

$\Rightarrow$ synthesis of LPR scalars

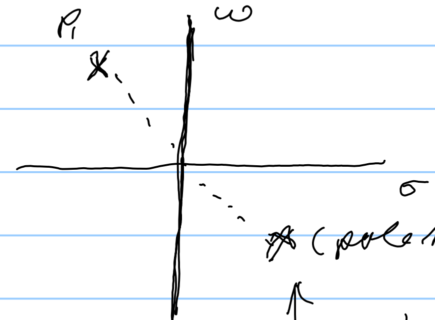
properties a) all singularities are poles

b) all poles (& zeros) are on $j\omega$ axis

c) & simple with positive residue

d) $y(1) \geq 0$

all points where $y(s) \to \infty$ are poles if rational



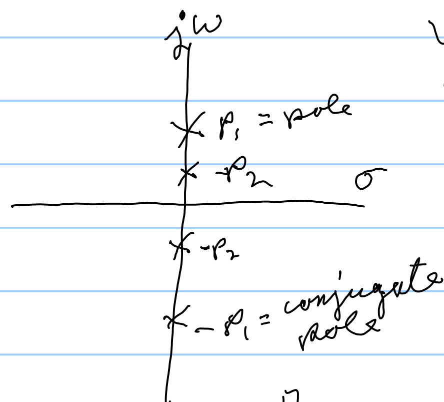
$y(s)\big|_{p_1}$ is singular $\quad y(-s) = y(s)$ (lossless)

$y(s)\big|_{-p_1}$ is singular

pole here if $p_1 = \sigma_1 + j\omega_1$ has $\sigma_1 < 0$

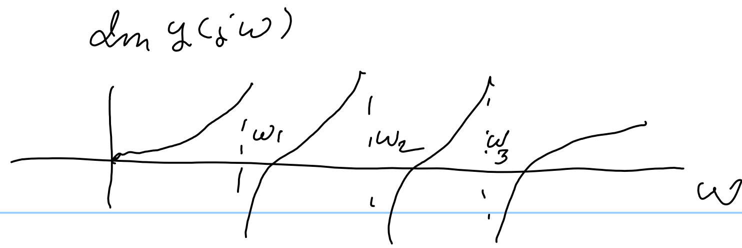
can't be if lossless

all poles are on $j\omega$ axis



$$y(s) \to \frac{k_1 s}{s^2 + p_1^2} \quad \& \text{ not } \quad \frac{\quad}{(s^2+p_1^2)^k}, \; k \geq 1$$

$$y(s) = \sum_{i=1}^{p} \frac{2 k_i s}{s^2 + \omega_i^2} + \frac{k_0}{s} + k_\infty s \quad ; \quad k_i > 0$$



$$\frac{d\, y(j\omega)}{d\omega} = \frac{d}{d\omega}\left[\sum_{i=1}^{p} \frac{2 j k_i \omega}{(-\omega^2+\omega_i^2)} + \frac{k_0}{j\omega} + k_\alpha j\omega\right]$$

Homework 3 due Tu 10/04/16