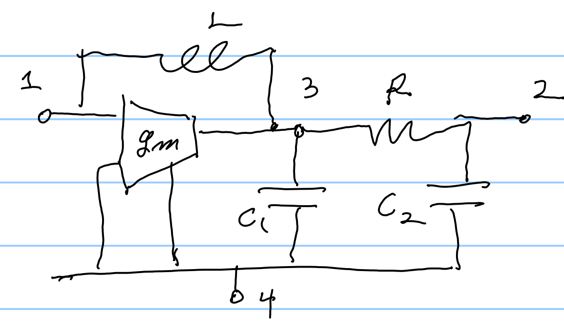
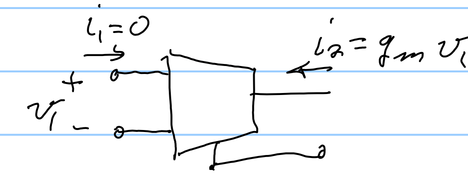
Note Title 9/22/2016

EE610
09/22/16

Indefinite admittance



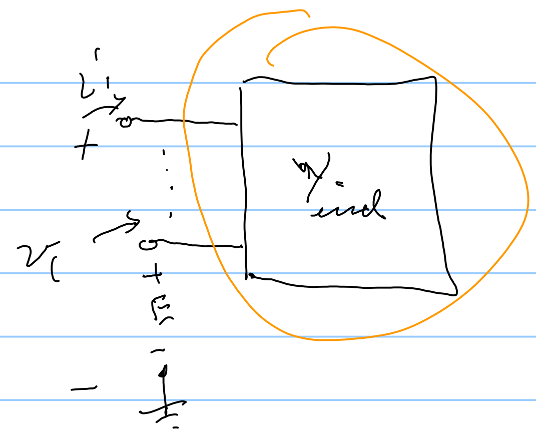


$$i_1 = \frac{1}{sL}(v_1 - v_3), \quad i_2 = G(v_2 - v_3) + sC_2(v_2 - v_4)$$

$$i_3 = \frac{1}{sL}\cdot(v_3 - v_1) + g_m(v_1 - v_4) + sC_1(v_3 - v_4) + G(v_3 - v_2)$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{sL} & 0 & -\frac{1}{sL} & 0 \\ 0 & G+sC_2 & -G & -sC_2 \\ -\frac{1}{sL}+g_m & -G & \frac{1}{sL}+sC_1+G & -g_m-sC_1 \\ -g_m & -sC_2 & -sC_1 & g_m+s(C_1+C_2) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \Rightarrow Y_{ind} \Rightarrow$$

Σ of entries in a row $= 0$
Σ of entries in a column $= 0$



$\Sigma i's = 0$ by KCL $\quad i = Y(v + E\underline{1})$ if n nodes

$\uparrow$ vector $\begin{bmatrix} E \\ E \\ E \\ E \\ \vdots \\ E \end{bmatrix}$

If ground node 4 $\Rightarrow v_4 = 0$; $i_4 = i_1' + i_2' + i_3'$ can ignore

$$Y_{\substack{def \\ 3\times 3 \\ \text{''} \\ nodal}} = \begin{bmatrix} \frac{1}{sL} & 0 & -\frac{1}{sL} \\ 0 & G + sC_2 & -G \\ -\frac{1}{sL} + g_m & -G & \frac{1}{sL} + sC + G \end{bmatrix}$$

If eliminate internal node 3 $\Rightarrow i_3' = 0$, so find v_3 in terms of other voltages (v_1 & v_2) to $Y_{2\text{-port}}$

$$i_3 = \begin{bmatrix} -\frac{1}{sL}+g_m & -G \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} G+sC+\frac{1}{sL} \end{bmatrix}\begin{bmatrix} v_3 \end{bmatrix} = 0$$

$$-v_3 = \begin{bmatrix} G+sC+\frac{1}{sL} \end{bmatrix}^{-1}\begin{bmatrix} -\frac{1}{sL}+g_m & -G \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{sL} & 0 \\ 0 & G+sC_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} -\frac{1}{sL} \\ -G \end{bmatrix}\begin{bmatrix} G+sC+\frac{1}{sL} \end{bmatrix}^{-1}\begin{bmatrix} \frac{1}{sC}+g_m & -G \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$Y_{2port} = \begin{bmatrix} \frac{1}{sL} & 0 \\ 0 & G+sC_2 \end{bmatrix} - \frac{1}{G+sC+\frac{1}{sL}}\begin{bmatrix} -\frac{1}{sL}(\frac{1}{sL}+g_m) & -\frac{1}{sL}(-G) \\ (-G)(\frac{1}{sL}+g_m-G) & (-G)(-G) \end{bmatrix}$$

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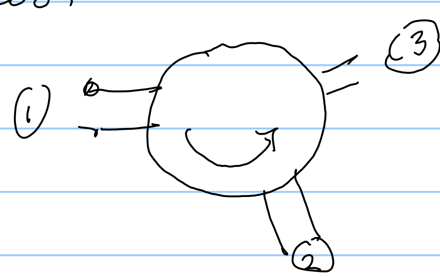
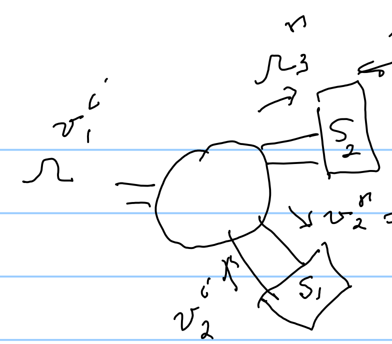
circulator:



$v_2^r = v_1^i$

$v_3^r = v_2^i$

$v_1^r = v_3^i$



$v_3^i = S_2 \cdot v_3^r$

$v_2^r = v^i_{load\,S_1} \Rightarrow v_2^i = S_1 \cdot v^i_{load\,S_1} = S_1 v_2^r$

$$v_1^r = v_3^i = S_2 v_3^r = S_2 \cdot v_2^i = S_2 \cdot S_1 v_2^r = S_2 \cdot S_1 v_1^i$$

$$\Rightarrow \quad S = S_2 . S_1$$

$\rightsquigarrow$ passive

$$\int_{-\infty}^{\infty} v^{i,T}(t)\, v^i(t)\, dt - \int_{-\infty}^{\infty} v^{r,T}(t)\, v^r(t)\, dt \geq 0 \text{ if passive}$$

Parseval's theorem

$$\int_{-\infty}^{\infty} f^*(t)\, g(t)\, dt = \int_{-\infty}^{\infty} F^*(j\omega)\, G(j\omega)\, \frac{d\omega}{2\pi}$$

$F(j\omega) =$ Fourier transform of $f(t)$

$$\int_{-\infty}^{\infty} v^{iT*}(t) \cdot v^{i}(t)\, dt = \int_{-\infty}^{\infty} V^{iT*}(j\omega)\, V^{i}(j\omega)\, \frac{d\omega}{2\pi}$$

$$\underbrace{\;- \int_{-\infty}^{\infty} v^{rT*}(t)\, v^{r}(t)\, dt\;}_{\geq 0 \text{ if passive}} \Rightarrow \underbrace{\;- \int_{-\infty}^{\infty} V^{rT*}(j\omega)\, V^{r}(j\omega)\, d\omega/_{2\pi}\;}_{\geq 0 \text{ if passive.}} \Big\} = - \int V^{i*T}(j\omega)\, S^{*T}(j\omega) S(j\omega) V^{i}(j\omega)$$

$$\int_{-\infty}^{\infty} V^{iT*}(j\omega) \left[1_n - S^{T*}(j\omega) S(j\omega) \right] V^{i}(j\omega) \frac{d\omega}{2\pi} \geq 0 \text{ if passive}$$

$\Rightarrow$ $1_n - S^{T*}(j\omega) S(j\omega)$ is positive (semi)-definite Hermitian form

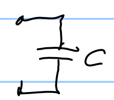
$\Rightarrow$ for real circuits $S^{*}(j\omega) = S(-j\omega)$

no poles of $S(s)$ in $\sigma > 0$ where $s = \sigma + j\omega$, $\sigma > 0$ means the right half plane

For lossless $1_n - S^{T*}(j\omega)S(j\omega) \equiv 0_n$ on all of $j\omega$ axis, $-\infty \le \omega \le \infty$

can extend to whole s-plane $\Rightarrow$

lossless $1_n = S^T(-s)S(s) \Leftrightarrow S^{-1}(s) = S^T(-s)$

Ex: 

$$S = (1+Y)^{-1}(1-Y) = (1+sC)^{-1}(1-sC) = \frac{1-sC}{1+sC}$$

$$S^{-1} = \frac{1+sC}{1-sC} = S(-s)$$

Bounded-real

1) $S^*(s) = S(s^*)$ in $\sigma > 0$ (a real circuit)
2) No singularities in $\sigma > 0$ (stability)
   (no poles on $j\omega$ axis)
3) $1_n - S^{T*}(j\omega)S(j\omega)$ is positive semidefinite (passive)

if $S(s)$ is rational & bounded-real $\Rightarrow$ BR & we can design a passive circuit

$$\int_{-\infty}^{\infty}\left[\left(\frac{V(j\omega)+I(j\omega)}{2}\right)^{T*}\left(\frac{V(j\omega)+I(j\omega)}{2}\right) - \left(\frac{V(j\omega)-I(j\omega)}{2}\right)^{T*}\left(\frac{V(j\omega)-I(j\omega)}{2}\right)\right]\frac{d\omega}{2\pi} \geq 0$$

$$\int_{-\infty}^{\infty} \left[I^{T*} V + V^{T*} I \right] - \left[- I^{T*} V - V^{T*} I \right] \frac{d\omega}{4} \times \frac{1}{2\pi} \geq 0$$

$$\int \left(V^{T*} Y^{*T} \cdot V + V^{T*} Y V \right) \frac{d\omega}{4\pi} \geq 0$$

$$\int V^{T*} \left[Y^{T*}(j\omega) + Y(j\omega) \right] V \cdot \frac{d\omega}{4\pi} \geq 0 \text{ passive}$$

$\Rightarrow$ $Y(j\omega) + Y^{T}(-j\omega)$ is positive semidefinite for all ω for which it exists

$\Rightarrow$ extend to $s \Rightarrow$

Lossless $Y(s) + Y^{T}(-s) = 0_n$

Ex: $Y(s) = sC$ $\qquad sC + (-s)C = 0$