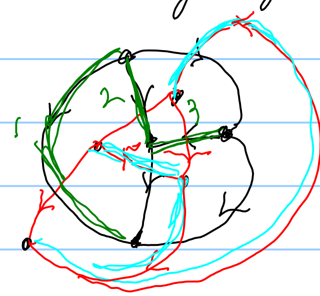
Note Title 9/20/2016

EE610
09/20/16

Dual $v \to i^d$, $i \to v^d$

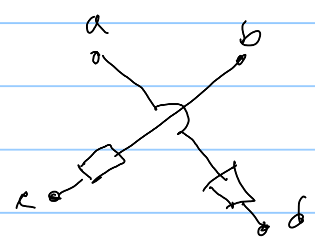
$i = c\frac{dv}{dt} \Rightarrow v = c\,di/dt \Rightarrow C \Rightarrow L \Rightarrow$ inductor

Dual graph $\Rightarrow$ planar graphs
technique



— = dual graph
— tree of dual
— Tree of original

If unavoidable crossing wires (branches) no graph dual



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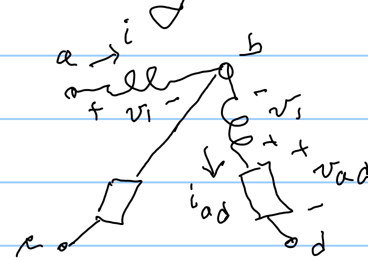


$\Rightarrow$ allows for planar graphs with no change in equations

Scattering matrix; $Av = Bi$; $2v^i = v + i$, $2v^n = v - i$

dual of gyrator
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow i = Yv$$

$$\Downarrow$$

$$\begin{bmatrix} v_1^d \\ v_2^d \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -g^d \\ g^d & 0 \end{bmatrix}}\begin{bmatrix} i_1^d \\ i_2^d \end{bmatrix} \Rightarrow v = Z^d i \qquad g^d = r$$

$$\begin{bmatrix} i_1^d \\ i_2^d \end{bmatrix} = Y^d \begin{bmatrix} v_1^d \\ v_2^d \end{bmatrix} \Rightarrow (Z^d)^{-1} = Y^d$$

$$Z^d = \begin{bmatrix} 0 & -g^d \\ g^d & 0 \end{bmatrix} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}, \quad (Z^d)^{-1} = \frac{1}{r^2}\begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} = Y^d = \begin{bmatrix} 0 & 1/r \\ -1/r & 0 \end{bmatrix}$$

$$v = v^i + v^n, \quad i = v^i - v^n \quad = \quad A(v^i + v^n) = B(v^i - v^n)$$

$$(A+B)v^n = (B-A)v^i \quad ; \quad v^n = S v^i \Rightarrow S = (A+B)^{-1}(B-A)$$

$$i = Yv \Rightarrow \quad A = Y, \; B = 1_m \quad ; \quad S = (1_m + Y)^{-1}(1_m - Y)$$

$$(1_m + Y)S = (1_m - Y) \Rightarrow \quad S + YS = 1_m - Y \Rightarrow \quad S - 1_m = -YS - Y$$

$$= -(Y)(S + 1_m)$$

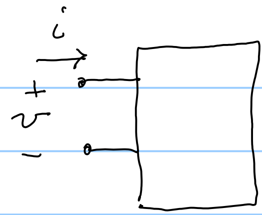
$$Y = (1_m - S)(1_m + S)^{-1}$$

$$(1_m + M)^{-1}(1_m - M) \overset{?}{=} (1_m - M)(1_m + M)^{-1}$$

$$(1_m - M)(1_m + M) \overset{?}{=} (1_m + M)(1_m - M)$$

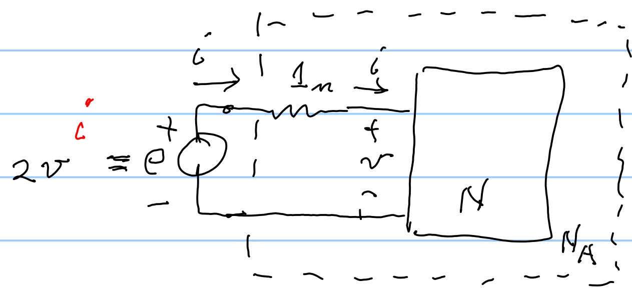
$$(1_m + M - M - M^2) \equiv (1_m + M - M - M^2)$$

Passivity:



$$P(t) = v^T(t) \cdot i(t)$$

$$\mathcal{E}(t) = \int_{-\infty}^{t} p(\tau)\,d\tau = \int_{0}^{t} v^T(\tau)\,i(\tau)\,d\tau + \mathcal{E}(0) \geq 0 \text{ defines passive}$$



$$2v^r = v - i, \quad 2v^i = v + i$$

$$v = v^i + v^r$$

$$i = v^i - v^r$$

$$v^T i = (v^i + v^r)^T (v^i - v^r)$$

$$= (v^{iT} + v^{rT})(v^i - v^r)$$

$$= v^{iT} \cdot v^i \underbrace{- v^{iT} v^r + v^{rT} v^i}_{=0} - v^{rT} v^r$$

$$p(t) = \underbrace{v^{iT} \cdot v^i}_{\text{incident}} - \underbrace{v^{rT} \cdot v^r}_{\text{reflected}} \Big\} \text{ powers}$$

$$\mathcal{E}(t) = \int_{-\infty}^{t} \left[v^{iT}(\tau) \cdot v^{i} - v^{rT}(\tau) v^{r}(\tau) \right] d\tau$$ if ≥ 0 then incidents dominate reflected

If $v^{i} = e/2$ is finite energy incident then for a passive circuit (N = network) Then The reflected energy is finite

Let
$$\int_{-\infty}^{t} \left[v^{iT}(\tau) v^{i}(\tau) \right] d\tau = L_{2\,norm} \text{ (up to time } t) = \|v^{i}\|^{2} \Big|_{-\infty}^{t}.$$

can chop $v^{iT}(t)$ at t_0 & $\int_{-\infty}^{t_0} = \int_{-\infty}^{t_0} + 0 = \int_{-\infty}^{\infty}$

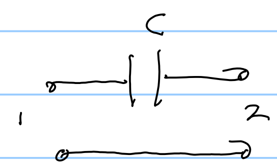
$$\|v^{i}\|_{L_2}^{2} \geq \|v^{r}\|_{L_2}^{2} \geq 0 \quad \text{for a passive circuit}$$

If S exists, true if N is passive, Then $i = Y_A \cdot e = Y_A (2v^{i})$

$$v - i = 2v^{r} = (e - i) - i = e - 2i = 2v^{r}$$
$$= e - 2Y_A e = (1_n - 2Y_A)\underset{\substack{\parallel \\ 2v^{i}}}{e} = 2v^{r}$$

$$v^r = (1_n - 2Y_A)\,v^i \Rightarrow S = 1_n - 2Y_A$$ } exists if N is passive

Ex:



$$Y = \begin{bmatrix} \alpha c & -\alpha c \\ -\alpha c & \alpha c \end{bmatrix} ; \quad S = (1_n + Y)^{-1}(1_n - Y)$$

$$1_2 + Y = \begin{bmatrix} 1+\alpha c & -\alpha c \\ -\alpha c & 1+\alpha c \end{bmatrix}, \quad (1_2 + Y)^{-1} = \frac{1}{\underbrace{(1+\alpha c)^2 - (\alpha c)^2}_{1+2\alpha c}} \begin{bmatrix} 1+\alpha c & \alpha c \\ \alpha c & 1+\alpha c \end{bmatrix}$$

$$1_n - Y = \begin{bmatrix} 1-\alpha c & \alpha c \\ \alpha c & 1-\alpha c \end{bmatrix}$$

$$S(\alpha) = \frac{1}{1+2\alpha c} \begin{bmatrix} 1+\alpha c & \alpha c \\ \alpha c & 1+\alpha c \end{bmatrix} \begin{bmatrix} 1-\alpha c & \alpha c \\ \alpha c & 1-\alpha c \end{bmatrix} = \frac{1}{1+2\alpha c} \begin{bmatrix} 1 & 2\alpha c \\ 2\alpha c & 1 \end{bmatrix}$$

$$S^T(-\alpha) = \frac{1}{1-2\alpha c} \begin{bmatrix} 1 & -2\alpha c \\ -2\alpha c & 1 \end{bmatrix} ; \quad S(\alpha) S^T(-\alpha) = \frac{1}{1-4\alpha^2 c^2} \begin{bmatrix} 1-4\alpha^2c^2 & 0 \\ 0 & 1-4\alpha^2c^2 \end{bmatrix} = 1_2$$

Lossless $\Leftrightarrow \mathcal{E}(\infty) \equiv 0$ & passive