Note Title 9/10/2016

EE 610

09/13/16

$$\left.\begin{aligned} E\frac{dx}{dt} &= Ax + Bu \\ y &= Cx \end{aligned}\right\}$$

If E is nonsingular E^{-1} exists, $\underbrace{\begin{aligned} \frac{dx}{dt} &= E^{-1}Ax + E^{-1}Bu \\ y &= Cx \end{aligned}}_{\text{state variable eqs}}$

$$sEx = Ax + Bu$$
$$y = Cx$$

$y(s) = T(s)u(s) \Leftarrow (sE - A)x = Bu$, $y = Cx$

$x = (sE - A)^{-1}Bu$

$y = C(sE - A)^{-1}Bu$, $T(s) = C(sE - A)^{-1}B$ = transfer function matrix

If $E = 1_n$ then $T(s) = C(s1_n - A)^{-1}B \to 0$ if $s \to \infty$

If a state eq: $sx = Ax + Bu$, $y = Cx + Du$ to handle resistors

$+ Eu' + \cdots$

But semi-state equations can give $T(s) = s^p + \cdots$, p any integer

Reason:

$$sNx = 1_k x + Bu, \quad y = Cx$$

$$-(1_k - sN)x = Bu, \quad x = -(1_k - sN)^{-1} Bu$$

$$(1_k - sN)^{-1} = \sum_{k=0}^{\infty} (sN)^k \Rightarrow 1_k \overset{?}{=} (1_k - sN)\left(\sum_{k=0}^{\infty} (sN)^k\right)$$

$$= \sum_{k=0}^{\infty} (sN)^k - \sum_{k=1}^{\infty} (sN)^k$$

$$= (sN)^0 = 1_k$$

if $N^{p+1} = 0_k$ but $N^p \neq 0$

then N is called nilpotent

then $(1_k - sN)^{-1} = \sum_{k=0}^{p} (sN)^k$

$\therefore \quad \alpha N x = 1_k x + Bu, \quad y = Cx$

then $T(\alpha) = C(\alpha N - 1_k)^{-1} B = C(-1)\left(\sum_{k=0}^{\nu} (\alpha N)^k\right) B$

gives $T(\alpha)$ to be polynomial in α

Ex: $N = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad N^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\alpha \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u, \quad y = [c_1 \ c_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$T(\alpha) = [c_1 \ c_2]\begin{bmatrix} -1 & \alpha \\ 0 & -1 \end{bmatrix}^{-1}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = [c_1 \ c_2]\begin{bmatrix} -1 & -\alpha \\ 0 & -1 \end{bmatrix} \cdot \frac{1}{1} \cdot \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$

choose $c_1 \neq 0, c_2 = 0, B_1 = 0, B_2 \neq 0$

$= [c_1 \ 0]\begin{bmatrix} -1 & -\alpha \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ B_2 \end{bmatrix} = -\alpha c_1 B_2$

$= T(\alpha)$

$$\alpha E x = Ax + Bu \quad , \quad y = Cx$$

$$\alpha P E x = PAx + PBu \quad , \quad y = Cx \quad , \quad P \text{ nonsingular}$$

$$x = Q\hat{x} \quad , \quad Q \text{ nonsingular}$$

$$\alpha PEQ\hat{x} = PAQ\hat{x} + PBu \quad , \quad y = CQ\hat{x}$$

$$\alpha \hat{E}\hat{x} = \hat{A}\hat{x} + \hat{B}u \quad , \quad y = \hat{C}\hat{x} \quad , \quad \hat{E} = PEQ, \ \hat{A} = PAQ, \ \hat{B} = PB$$

$$\hat{C} = CQ$$

$$T(\alpha) = C(\alpha E - A)^{-1}B \quad , \quad \hat{T}(\alpha) = \hat{C}(\alpha\hat{E} - \hat{A})^{-1}\hat{B} = CQ(\alpha PEQ - PAQ)^{-1}PB$$

$$= CQ(P(\alpha EQ - PAQ)^{-1}PB$$

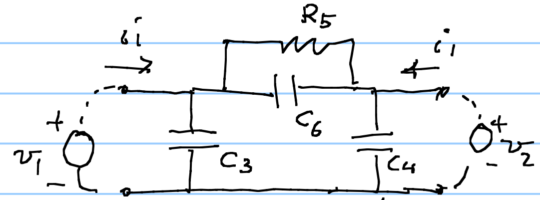
$$= CQ(P[\alpha EQ - AQ])^{-1}PB$$

$$= CQ(P(\alpha E - A)Q)^{-1}PB$$

$$= CQQ^{-1}(\alpha E - A)^{-1}P^{-1} \cdot PB$$

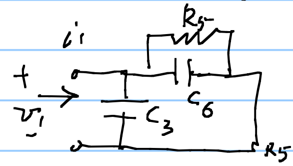
$$= C(\alpha E - A)^{-1}B \equiv T(\alpha)$$

Look at admittance matrix as $T(s)$, $2\times 2 = 2$-port



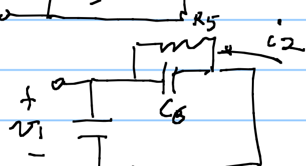
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y(s) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$$



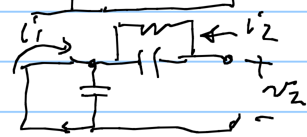
$$y_{11} = sC_3 + sC_6 + G_5\,,\quad G_5 = 1/R_5$$

$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$$



$$\frac{i_2}{v_1} = -(sC_6 + G_5) = y_{21}$$

$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0}$$

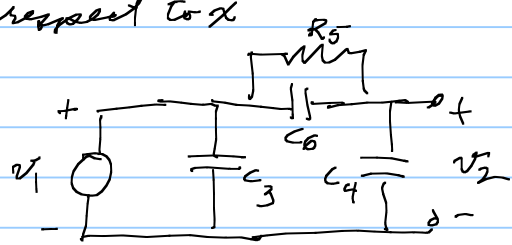


$$y_{12} = y_{21} = -(sC_6 + G_5)$$

$$y_{22} = sC_4 + sC_6 + G_5 \text{ by symmetry}$$

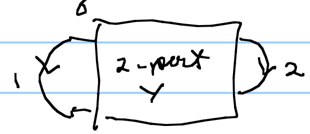
$$Y = \begin{bmatrix} s(C_3+C_6)+G_5 & -sC_6-G_5 \\ -sC_6-G_5 & s(C_4+C_6)+G_5 \end{bmatrix} = Y_{2\text{-port}}$$

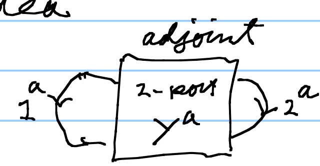
To sensitivity of $T(s)$ with respect to x $\quad S_x^{T(s)} = \dfrac{dT(s)/dx}{T(s)/x}$



$$T(s) = \frac{v_2}{v_1} = \frac{1/sC_4}{\frac{1}{sC_4} + \frac{1}{sC_6+G_5}}$$

derive $S_{C_6}^{v_2/v_1}$ use adjoint idea





use these 2 to find $\partial T/\partial x$ by analyzing

assume the adjoint has the same graph $v_b^T i_b = v_b^{aT} i_b^a = 0$

$\Rightarrow v_b^T i_b^a = 0 \quad v_b^{aT} i_b = 0$; $i_{b-2} = Y_{b-2 \times b-2} \cdot v_{b-2}$

$v_b^T i_b^a - v_b^{aT} i_b = 0 \qquad i_{b-2}^a = Y_{b-2 \times b-2} \cdot v_{b-2}^a$

$$v_{b_1} \cdot i_{b_1}^a + v_{b_2} \cdot i_{b_2}^a - v_{b_1}^a i_{b_1} - v_{b_2}^a i_{b_2} + v_{b-2}^T i_{b-2}^a - v_{b-2}^{aT} \cdot i_{b-2} = 0$$

$$\frac{\partial v_{b_1}}{\partial x} \cdot i_{b_1}^a + v_{b_1} \cdot \frac{\partial i_{b_1}^a}{\partial x} + \frac{\partial v_{b_2}}{\partial x} i_{b_2}^a + v_{b_2} \cdot \frac{\partial i_{b_2}^a}{\partial x} -$$