EE 610
09/08/16

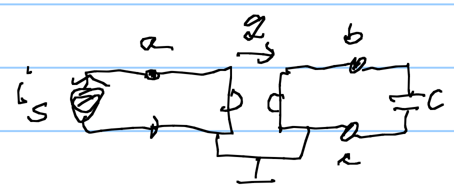
$$\underbrace{\left[\; A\mathcal{C}^T \;\vdots\; -B\mathcal{D}^T \right]}_{b \text{ columns}} \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = A\mathcal{E} - B\dot{j} = Mx, \quad x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}, \ b \text{ vector}$$

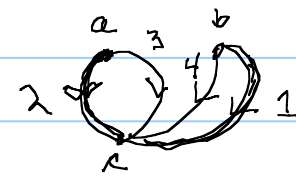
(b rows)

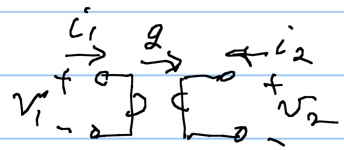
$A\mathcal{C}^T$ is $b \times t$ columns, $-B\mathcal{D}^T$ is b rows ℓ columns

gives b eqs in b unknowns

Example:







$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad g = \text{gyration conductance}$$

$$Av = Bi, \quad i = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \quad \begin{bmatrix} AC & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & g & 0 \end{bmatrix} v = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \dot{i}$$

$$\begin{bmatrix}0\\0\end{bmatrix} = \underbrace{\left[\begin{array}{cc|cc}1 & 0 & 0 & 1\\0 & 1 & 1 & 0\end{array}\right]}_{K}\dot{i}_b \Rightarrow 0 = \mathcal{E}\dot{i}_b, \qquad \begin{bmatrix}0\\0\end{bmatrix} = \underbrace{\left[\begin{array}{cc|cc}0 & -1 & 1 & 0\\-1 & 0 & 0 & 1\end{array}\right]}_{-K^T} v_b \Rightarrow 0 = \mathcal{J} v_b$$

$$\mathcal{E}^T = \begin{bmatrix}1 & 0\\0 & 1\\0 & 1\\1 & 0\end{bmatrix}, \quad \mathcal{J}^T = \begin{bmatrix}0 & -1\\-1 & 0\\1 & 0\\0 & 1\end{bmatrix} \quad \text{check} \quad \mathcal{E}\mathcal{J}^T = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$A\mathcal{E}^T = \begin{bmatrix}RC & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & -g\\0 & 0 & g & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\\0 & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}RC & 0\\0 & 0\\-g & 0\\0 & g\end{bmatrix}; \quad B\mathcal{J}^T = \mathbb{1}_4\begin{bmatrix}0 & -1\\-1 & 0\\1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}0 & -1\\-1 & 0\\1 & 0\\0 & 1\end{bmatrix}$$

$$\left[A\mathcal{E}^T \,\vdots\, -B\mathcal{J}^T\right] = \left[\begin{array}{cc|cc}RC & 0 & 0 & 1\\0 & 0 & 1 & 0\\-g & 0 & -1 & 0\\0 & g & 0 & -1\end{array}\right]; \quad \begin{bmatrix}v_1\\v_2\\\dot{i}_3\\\dot{i}_4\end{bmatrix}_b, \quad e = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \quad \dot{j} = \begin{bmatrix}0\\-\dot{i}_5\\0\\0\end{bmatrix}$$

$$Ae = \underline{0}_4, \quad B\dot{j} = \mathbb{1}_4 \cdot \dot{j} = [0\ -\dot{i}_5\ 0\ 0]^T \Rightarrow -B\dot{j} = [0\ \dot{i}_5\ 0\ 0]^T$$

$$\left[\begin{array}{cc:cc} sC & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \hdashline -g & 0 & -1 & 0 \\ 0 & g & 0 & -1 \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \end{bmatrix}_b = \begin{bmatrix} 0 \\ +i_s \\ 0 \\ 0 \end{bmatrix}$$

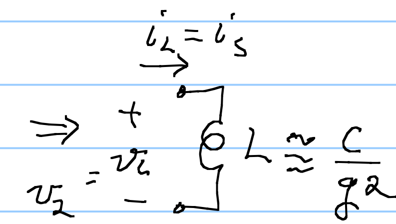
describe the circuit

$$1_2 \begin{bmatrix} i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -g & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_b \Rightarrow \begin{bmatrix} sC & 0 \\ 0 & 0 \end{bmatrix} v_t + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ +i_s \end{bmatrix}$$

$$\begin{bmatrix} sC & 0 \\ 0 & 0 \end{bmatrix} v_t + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -g & 0 \\ 0 & g \end{bmatrix} v_t = \begin{bmatrix} sC & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ +i_s \end{bmatrix}$$

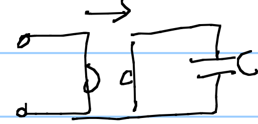
$$sC\, v_1 + g v_2 = 0 \Rightarrow v_1 = -\frac{g}{sC} v_2$$

$$-g v_1 + 0\, v_2 = -i_s \Rightarrow \frac{g^2}{sC} v_2 = +i_s \Rightarrow$$



$$g^2 v_L = sC\, i_L \Rightarrow v_L = s\left(\frac{C}{g^2}\right) i_L$$

$L = \frac{C}{g^2}$ $\equiv$



g C

if $g = 1/r = 10^{-3}$

$C = 1\,\mu fd \Rightarrow L = \frac{C}{g^2} = \frac{10^{-6}}{10^{-6}} = 1\,\text{Hy}$

$$\left[\begin{array}{cc:cc} sC & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \hline -g & 0 & -1 & 0 \\ 0 & g & 0 & -1 \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \end{bmatrix}_b = \begin{bmatrix} 0 \\ +i_s \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} sC & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ g & 0 & 1 & 0 \\ 0 & -g & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} i_s$$

$$\begin{bmatrix} E \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} \mathcal{A} \end{bmatrix} x + \begin{bmatrix} \mathcal{B} \end{bmatrix} u \quad , \quad u = \text{input}$$

$E = \begin{bmatrix} C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is singular

semi-state equations

y = output, u = input

x = semi-state

$y = \begin{bmatrix} \mathcal{C} \end{bmatrix} x$ if y is a linear combination of voltages & currents in the circuit

$$\left.\begin{aligned} E\frac{dx}{dt} &= Ax + Bu \\ y &= Cx \end{aligned}\right\}$$ generic semi-state eqn.
differential-algebraic eqs.