Solutions 303H F2016 final

#1. a) as $I_{D_Q} = 1\,ma$, $v_o\big|_Q = RI_D = 10^3 \times 10^{-3} = 1V \Rightarrow V_{DS_Q} = V_{DD} - V_o = 6-1 = 5V = V_{DS_Q}$, $V_{GS_Q} = V_{GG} - V_o = 3-1 = 2V = V_{GS_Q}$

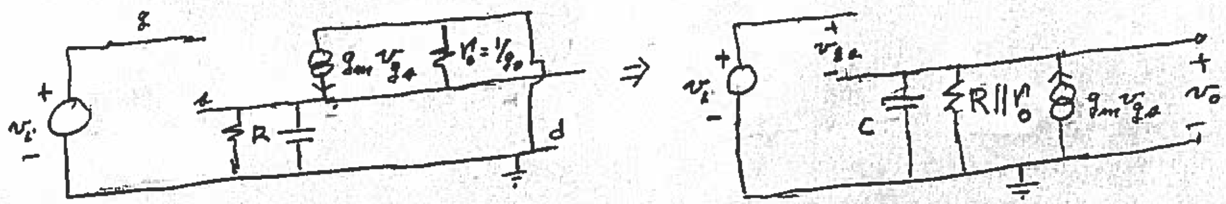
as $V_{DS_Q} = 5 > V_{GS_Q} - V_{TO} = 2-1 = 1 \Rightarrow$ saturation

b) as $I_{D_Q} = 1ma = 10^{-3} = k(V_{GS_Q} - V_{TO})^2(1+\lambda V_{DS_Q}) = k(1)^2(1+\frac{1}{10}\times 5) = k \times 1.5 \Rightarrow k = \frac{10^{-3}}{1.5} = \frac{2}{3}\times 10^{-3}\,A/V^2$

c) $g_m = \frac{\partial i_D}{\partial v_{GS}}\Big|_Q = 2k(V_{GS}-V_{TO})(1+\lambda V_{DS})\Big|_Q = 2\cdot\frac{10^{-3}}{1.5}(1)^2(1.5) = 2\times 10^{-3} = g_m$

$g_o = \frac{\partial i_D}{\partial v_{DS}}\Big|_Q = k(V_{GS}-V_{TO})^2\cdot\lambda = \frac{10^{-3}}{1.5}(1)^2\cdot\frac{1}{10} = \frac{1\times 10^{-4}}{1.5} = \frac{2}{3}\times 10^{-4}\,\mho = 0.667\times 10^{-4} = 0.067\times 10^{-3} = g_o$

d) as $C_{gs} = C_{gd} = 0$ the only capacitor is C; d → gnd as $v_{dd} = 0$



e) α) $v_{gs} = v_i - v_o$ by KVL, β) $C\dot{v}_o + G_o v_o - g_m v_{gs} = 0$ by KCL; $G_o = G + g_o$, $G = 1/R$

α)→β) ⇒ $C\dot{v}_o + G_o v_o - g_m v_i + g_m v_o \Rightarrow C\dot{v}_o + G_o v_o + g_m v_o = g_m v_i$, let $\dot{} = s = d/dt$

$$\frac{v_o}{v_i} = \frac{g_m}{sC + G_o + g_m} = \frac{2\times 10^{-3}}{10^{-6}s + (10^{-3} + 0.067\times 10^{-3} + 2\times 10^{-3})} = \frac{2\times 10^{3}}{s + 3.067\times 10^{3}}$$

(note $\frac{v_o}{v_i}(0)$ = DC gain $= 2/3.067 < 1$)

#2. $v_o(0_-) = \frac{V_{DD}}{2} \Rightarrow v_i(0_+) = 0 \Rightarrow v_{GS_{M_n}}(0_+) = 0 \Rightarrow M_n$ off

$= v_o(0_+) = 3$

$$\Rightarrow v_{SG_{M_p}} = V_{DD} \quad \& \quad v_{SD_{M_p}} = V_{DD} - v_o(0_+) = \frac{V_{DD}}{2} = 3$$

$$\Rightarrow v_{SG_p} - |V_{TOp}| = V_{DD} - \frac{1}{6}V_{DD} = \frac{5}{6}V_{DD} \gg \frac{V_{DD}}{2} = V_{SD_{M_p}} = 3 \Rightarrow M_p = \text{ohmic}$$

as v_o increases with t, $V_{SD} = V_{DD} - v_o$ decreases ~~while~~ $V_{SG} = V_{DD} \Rightarrow$ ohmic in $t_0 = 1$

$$i_S = k_p\left[2(v_{SG} - |V_{TOp}|)v_{SD} - v_{SD}^2\right] = k_p\left[2\left(V_{DD} - \frac{1}{6}V_{DD}\right)(V_{DD} - v_o) - (V_{DD} - v_o)^2\right]$$

$\therefore$ let $x = V_{DD} - v_o$, $x(0_+) = \frac{1}{2}V_{DD}$

$$i_S = C\frac{dv_o}{dt} = C\frac{d(v_o - V_{DD})}{dt}, \text{ as } \frac{dV_{DD}}{dt} \equiv 0$$

$x(0) = V_{DD}/2 = 3$

at $x = V_{DD} - v_o \Rightarrow x(t_5) = V_{DD} - 5 \equiv 1\,V$

$$= -C\,dx/dt$$

$$\equiv k_p\left[2\left(\frac{5}{6}V_{DD}\right)x - x^2\right] \Rightarrow \frac{dx}{dt} + \frac{k_p}{C}\left[\frac{10}{6}V_{DD}x - x^2\right] = 0 = \frac{dx}{dt} + \frac{k_p}{C}x(10 - x) = 0$$

$$\Rightarrow \frac{dx}{x(x-10)} = \frac{k_p}{C}dt \Rightarrow \int_{x(0)}^{x(t)} \frac{dX}{X(X-10)} = \int_0^t \frac{k_p}{C}d\tau = \frac{k_p}{C}t$$

Left side, use $\frac{1}{X(X-10)} = \frac{k_0}{X} + \frac{k_1}{X-10}$; $k_0 = \frac{x}{x(x-10)}\Big|_{X=0} = -1/10$ $k_1 = \frac{1}{X}\Big|_{X=10} = 1/10$

$$= -\frac{1/10}{X} + \frac{1/10}{X-10}$$

$$\Rightarrow \int_{x(0_+)}^{x(t)} \frac{1}{10}\left[-\frac{1}{X}dX + \frac{1}{X-10}dX\right] = \frac{1}{10}\left[-\ln X + \ln(X-10)\right]\Big|_{x(0)}^{x(t)} = \frac{1}{10}\ln\left(\frac{|X-10|}{|X|}\right)\Big|_{x(0)=3}^{x(t)=1}$$

$$\therefore \frac{1}{10}\ln\left(\frac{|x(t)-10|}{|x(t)|}\cdot\frac{|x(0)|}{|x(0)-10|}\right) = \frac{k_p}{C}t \Rightarrow \left|\frac{x(t)-10}{x(t)}\right| = \left|\frac{x(0)-10}{x(0)}\right| e^{10\frac{k_p}{C}t} = \frac{7}{3}e^{10t}$$

$$\text{time to change} = t_5 = \frac{C}{k_p}\cdot\frac{1}{10}\ln\left(\left|\frac{1-10}{1}\right|\cdot\left|\frac{3}{7}\right|\right) = \frac{10^{-6}}{10^{-6}}\cdot\frac{1}{10}\ln\left(\frac{27}{7}\right) = \frac{1}{10}(1.35) = 0.135 \text{ sec.}$$

#3. a) Each branch has $y_j(s) = \dfrac{1}{R_j + \frac{1}{sC_j} + sL_j} = \dfrac{sC_j}{s^2 L_j C_j + R_j C_j s + 1} = \dfrac{\frac{1}{L_j} s}{s^2 + \frac{R_j}{L_j} s + \frac{1}{L_j C_j}}$ $\neq 0$ as R_j, C_j & $L_j > 0$

By KCL, the currents in these add to give the input current vs the input voltage which is the same for all

i.e.

a₁) $y(s) = \dfrac{I}{V} = \sum_{j=1}^{m} y_j \cdot \dfrac{V}{V} = \sum_{j=1}^{m} y_j(s) = \sum_{j=1}^{m} \dfrac{\frac{1}{L_j} s}{s^2 + \frac{R_j}{L_j} s + \frac{1}{L_j C_j}} = s \sum_{j=1}^{m} \dfrac{1/L_j}{s^2 + \frac{R_j}{L_j} s + \frac{1}{L_j C_j}}$

The poles are at the zeros of the denominator polynomials so

a₂) $p_j = -\dfrac{R_j}{2L_j} \pm \dfrac{1}{2}\sqrt{\left(\frac{R_j}{L_j}\right)^2 - 4 \cdot \frac{1}{L_j C_j}}$ $\quad j = 1, \cdots, m$

a₃) There is a zero at $s = 0$

b) $y_1 = \dfrac{s}{s^2 + 1} + \dfrac{s}{s^2 + \frac{1}{4}}$

$I(s) = y V(s)$; $V(s) = 1/s$ (for $v(t) = 1(t)$) $\Rightarrow I(s) = \dfrac{1}{s^2+1} + \dfrac{1}{s^2 + 1/4}$

Using s as Laplace transform variable, $\mathcal{L}[f(t)] = \dfrac{1}{s^2 + \omega_0^2} \Rightarrow f(t) = \dfrac{1}{\omega_0} \sin \omega_0 t$

$\Rightarrow$

$i(t) = (\sin t + \frac{1}{2} \sin 2t)1(t)$ which has $i(0_-) = i(0_+) = 0$