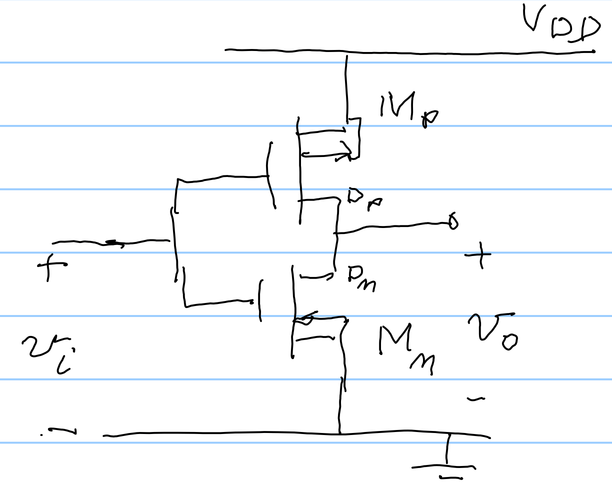
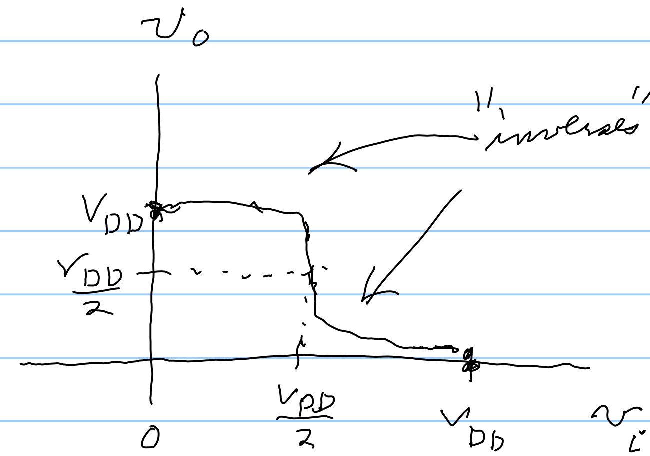
Note Title 10/27/2016

EE303N
10/27/16

CMOS inverter

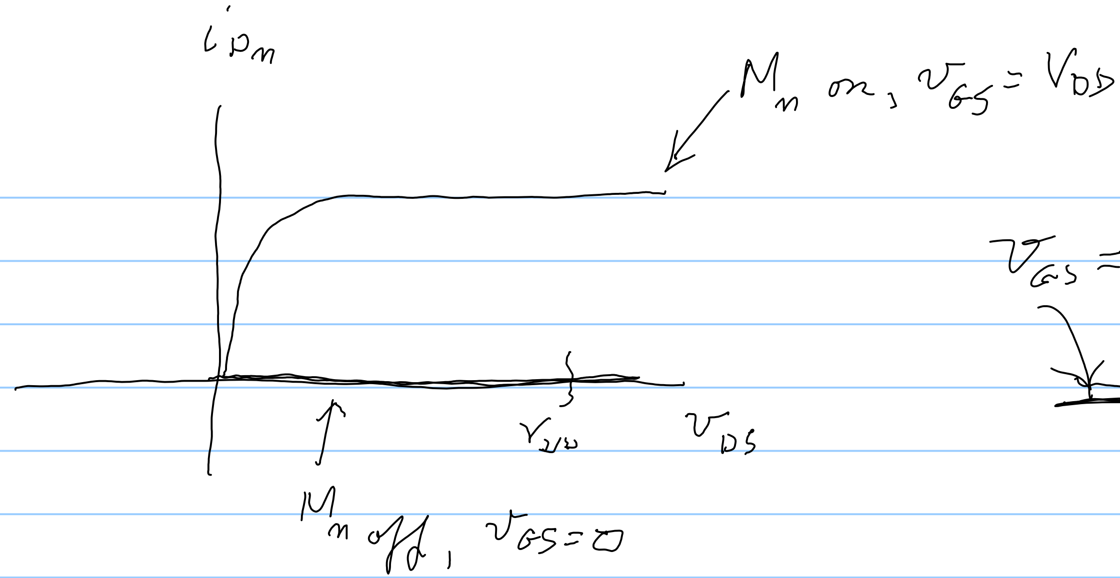


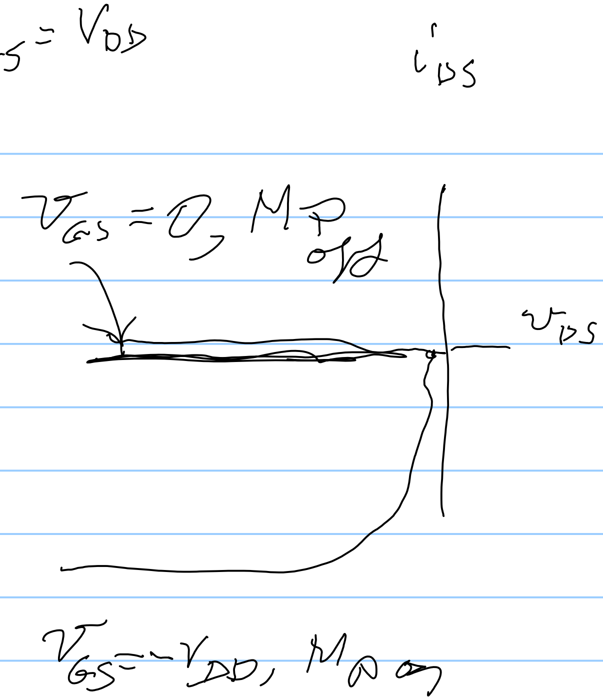


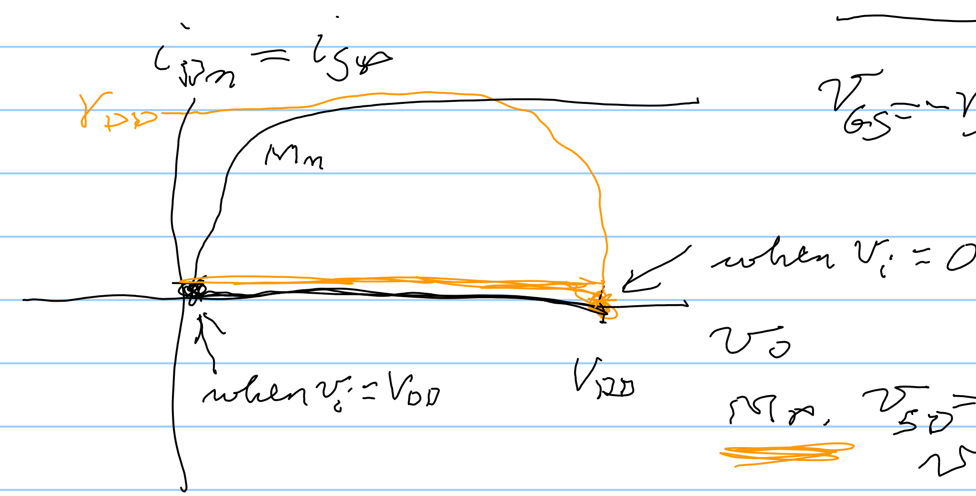
@ $v_i = 0$, $M_n = \text{off}$, $i_{D_n} = 0 \simeq i_{S_p} = -i_{D_p} = 0 \simeq v_{SD_p} = 0$ ∴ $v_o = V_{DD} - v_{SD_p}$
$= V_{DD}$

@ $v_i = V_{DD} \equiv$ binary 1, $M_p = \text{off}$, $i_{D_p} = 0 = -i_{D_n} \Rightarrow v_{DS_n} = 0 = v_o$ $\Rightarrow v_o \equiv 1$ binary
$\Rightarrow$ binary 0

No current flows when $v_o = 0$ or $V_{DD} \Rightarrow$ only uses power during transitions

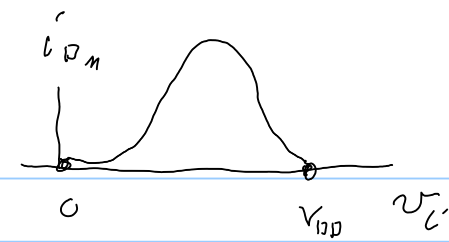






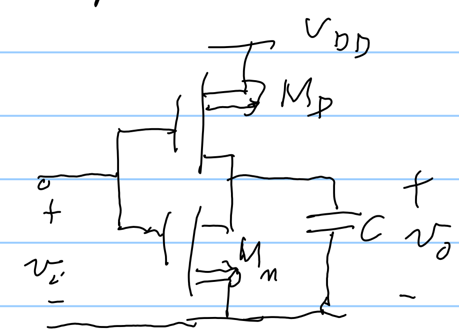
M_p, $v_{SD} = V_{DD} - v_O$

$v_D = V_{DD} - v_{SD}$



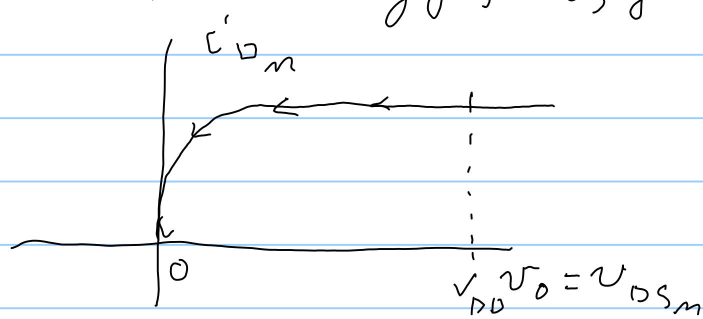
delivers power into the inverter when transition

Load with a capacitor



assume charged C to a "1"
then @ $t=0$, $v_o = V_{DD}$
(v_i for $t<0$ was a "0")
change v_i to V_{DD}

for $t = 0+$ M_p is off, v_{GS} for $M_n = V_{DD}$



@ large t

@ $t=0$ $\underbrace{\underset{V_{DD}}{\overset{\shortparallel}{v_{GS}}} - V_{TO} < \underset{V_{DD}}{\overset{\shortparallel}{v_{DS}}}}$

starts @ $t=0$ in saturation
$v_{GS} = V_{DD} - V_{TO} > v_{DS} = 0$
ends in ohmic region

$$i_{cap} = C\frac{dv_O}{dt} = -i_{D_n} = -\frac{KP'}{2}\frac{W}{L}\begin{cases}(V_{DD}-V_{TO})^2 & \text{in saturation} \\ (V_{DD}-V_{TO})v_O - \frac{1}{2}v_O^2 & \text{in ohmic region}\end{cases}$$

solution in saturation $\quad \frac{dv_o}{dt} = -\frac{KP}{C2}\frac{W}{L}(V_{DD}-V_{TO})^2$

$$v_o(t) = V_{DD} - \frac{\frac{KP}{2}\frac{W}{L}(V_{DD}-V_{TO})^2}{C}\int_0^t d\tau \qquad \text{for } V_{DD}-V_{TO} > v_o$$

$$-\left(\frac{v_o(t) - V_{DD}}{(KP/2)\frac{W}{LC}(V_{DD}-V_{TO})^2}\right) = t$$

desire the time t_0 when $v_o = V_{DD} - V_{TO}$
(for greater t M_n is ohmic & we need to solve the Riccati eq)

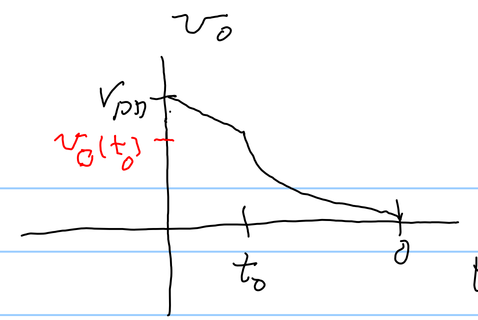
$$v_O(t_0) = V_{DD} - t_0\left[\frac{KP}{2}\frac{W}{LC}(V_{DD}-V_{TO})^2\right]$$

where

$$t_0 = -\frac{(V_{DD}-V_{TO}-V_{DD})C}{\frac{KP}{2}\frac{W}{L}(V_{DD}-V_{TO})^2} = \frac{2V_{TO}C}{KP\frac{W}{L}(V_{DD}-V_{TO})^2} \qquad \text{a small time}$$

after t_0 need to solve the Riccati eq.

$$C\frac{dv_O}{dt} = -\frac{KP}{2}\frac{W}{L}\left((V_{DD}-V_{TO})v_O - \frac{1}{2}v_O^2\right)$$



to proceed

$$\frac{dv_O}{(V_{DD}-V_{TO})v_O - \frac{1}{2}v_O^2} = -\frac{KP}{2C}dt\frac{W}{L} \quad \text{integrate}$$

$$\int_{v_O(t_0)}^{v_O(t)} \frac{dx}{ax - \frac{1}{2}x^2} = \int_{t_0}^{t} \frac{-KP}{2C}d\tau \qquad a = (V_{DD}-V_{TO})$$

desire a partial fraction expansion of $\frac{1}{ax-\frac{1}{2}x^2} = \frac{1}{ax\left(1-\frac{x}{2a}\right)}$

$$= \frac{-2a}{ax(x-2a)} = \frac{-2}{x(x-2a)} = \frac{k_0}{x} + \frac{k_a}{x-2a} = \frac{1/a}{x} + \frac{-1/a}{x-2a}$$

$$\text{w/ } k_0 = \frac{-2ax}{ax(x-2a)}\Big|_{x=0} = \frac{-2}{-2a}, \quad k_a = \frac{(x-2a)(-2)}{x(x-2a)}\Big|_{x=2a} = \frac{-2}{x}\Big|_{x=2a} = -1/a$$

$$\int_{v_0(t_0)}^{v(t)} \frac{(1/a)\,dx}{x} + \int_{v_0(t_0)}^{v_0(t)} \frac{-(1/a)}{x-2a}\,dx = \frac{\ln x}{a}\Big|_{v_0(t_0)}^{v_0(t)} - \frac{\ln}{a}\Big|_{v_0(t_0)-2a}^{v_0(t)-2a}$$

$$\frac{1}{a}\ln\left(\frac{v_0(t)\left(v_0(t_0)-2a\right)}{v_0(t_0)\left(v_0(t)-2a\right)}\right) = -\frac{K P_w}{2cL}\Big|_{t_0}^{t} = \frac{wKP}{L2c}t_0 - \frac{KP_w}{2cL}t$$

$$= \frac{v_0(t)\left(v_0(t_0)-2a\right)}{v_0(t_0)\left(v_0(t)-2a\right)} = e^{-a\frac{KP}{2}\frac{w}{L}t}$$

~~solve~~ solve for $v_0(t)$