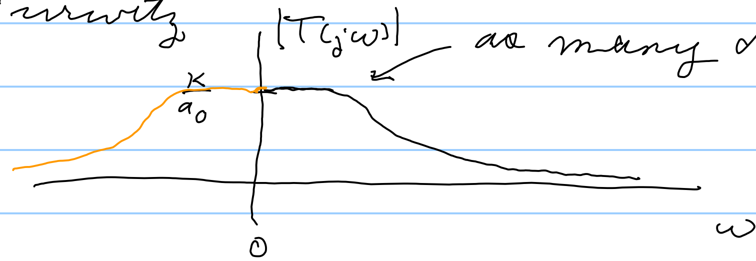
EE303H
10/25/16

Note Title 10/25/2016

Maximally flat low pass filter

$$T(s) = \frac{v_o}{v_i}(s) = \frac{K}{s^m + a_{m-1} s^{m-1} + a_1 s^1 + a_0} = \frac{K}{D(s)}$$

$D(s)$ = Hurwitz



as many derivatives = 0 @ $\omega = 0$

$$|T(j\omega)| = \frac{|K|}{|(j\omega)^m + \dots + a_1 j\omega + a_0|}$$

$$\frac{d\ 1/|T(\omega)|}{d\omega} = -\frac{1}{|T(j\omega)|^2} \cdot \frac{d|T(j\omega)|}{d\omega} = \frac{d}{d\omega}|(j\omega)^m + \dots + a_1 j\omega + a_0|$$

$$\frac{d|T(j\omega)|^2}{d\omega} = 2|T(j\omega)|\, d|T(j\omega)|/d\omega$$

$$\therefore \frac{d\, {}^1\!/\!|T(j\omega)|^2}{d\omega} = \frac{d}{d\omega}\left(\left((j\omega)^m + \dots + a_1 j\omega + a_0 \right)\left((-j\omega)^m + \dots + a_1(-j\omega) + a_0 \right) \right)$$

$$= \left(j^m \cdot m\omega^{m-1} + \dots a_1 j \right)\left((-j)^m \omega^m + \dots + a_1(-j\omega) + a_0 \right)$$

$$+ \left((j^m) \cdot \omega^m + \dots + j\omega a_1 + a_0 \right)\left((-j)^m m\omega^{m-1} + \dots - j a_1 \right)$$

$$= \cancel{b_0}^{=0} + b_1 \omega + b_2 \omega^2 + \dots + 2m\,\omega^{2m-1} \Rightarrow = b_1\omega + 2m\omega^{2m-1}$$

desire $b_0, \dots, b_{2m-2} = 0$ by choice of a's $\Rightarrow$ function of ω^2

$$0 = b_1\omega + 2m\,\omega^{2m-1} \Rightarrow \omega^{2m} = -b_0/2m = -1 \text{ if normalize}$$

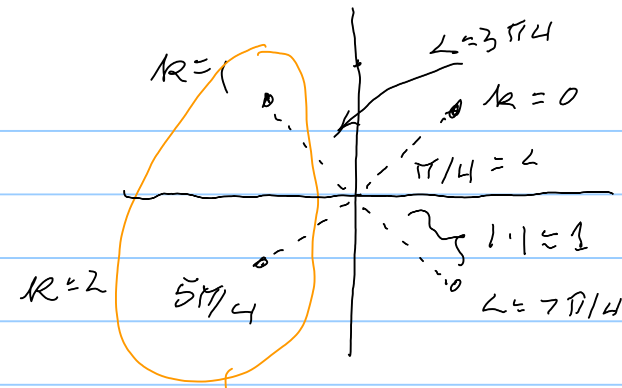
$\omega^{2m} = -1$ desire $2m$ roots of -1 complex numbers on unit circle

$$= e^{j\pi + j2k\pi} \qquad k = 0, 1, 2, \dots$$

$$\omega_k = e^{\frac{j(\pi + 2k\pi)}{2m}}$$

if $m = 2$, $\frac{\pi + 2k\pi}{4} = \pi\left(\frac{2k+1}{4}\right)$

$k = 0$, $\pi/4$, $k = 1$, $3\pi/4$, $k = 2$, $\frac{5\pi}{4}$, $k = 3$, $7\pi/4$, $k = 4$, $\frac{9\pi}{4} \approx \frac{\pi}{4}$



$-\cos 45^\circ + j\sin 45^\circ = \omega_1$

$(a^2+a^2) = 1 = 2a^2 \therefore a = \pm\sqrt{1/2}$

$a = \cos 45^\circ = \sin 45^\circ$

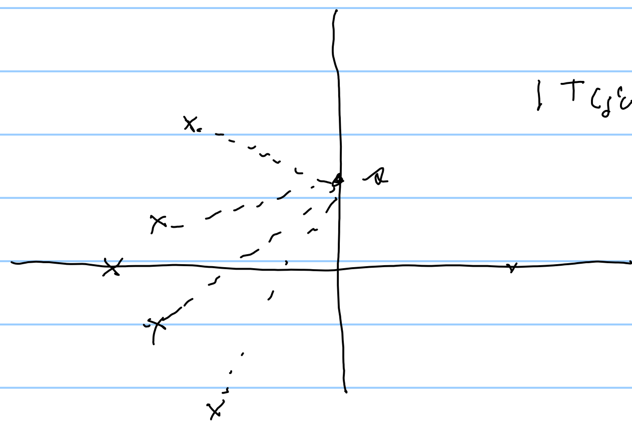
choose these to get an Hurwitz polynomial

$$\omega_1 = e^{j\frac{\pi+2\pi}{4}} = e^{j3\pi/4} = -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}, \quad \omega_2 = \omega_1^* = -\frac{1}{\sqrt{2}} - j^1/\sqrt{2}$$

$$D(s) = (s-(\omega_1))(s-(\omega_2)) = s^2 - (\omega_1+\omega_2) + \omega_1\omega_2$$
$$= s^2 - (-2/\sqrt{2})s + 1 = s^2 + \sqrt{2}\,s + 1$$

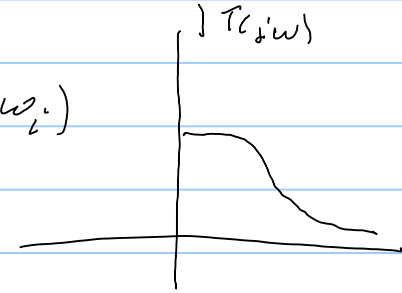
$$\therefore \quad T(s) = \frac{K}{s^2 + \sqrt{2}\,s + 1}$$ is maximally flat transfer function @ $\omega = 0$

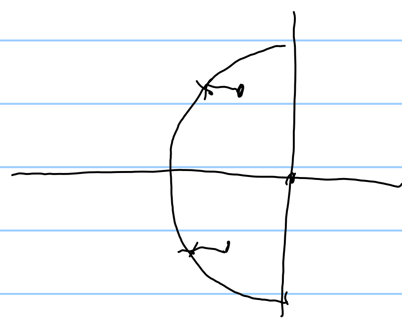
$$T(j\omega) = \frac{K}{(z-\omega_1)(z-\omega_1^*)(z-\omega_2)(z-\omega_2^* \cdots)}$$

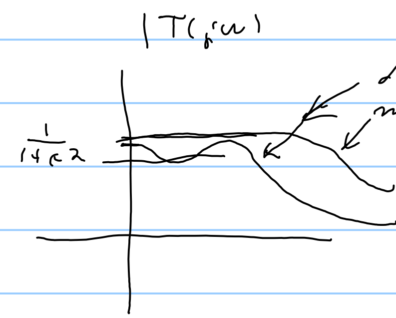


$$|T(j\omega)| = \frac{1}{\prod_{i=1}^{m} \text{distances } (j\omega - \omega_i)}$$

on unit circle







due to moving poles off the unit circle

max flat

If move by tanh ≈ 1/ε gives equal ripple