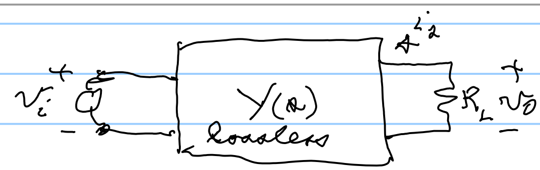
Note Title 10/17/2016

EE303H

10/20/16



$$\frac{v_o}{v_i}(s) = A(s)$$

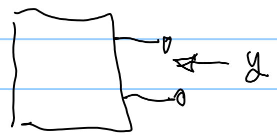
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad \begin{bmatrix} v_1 = v_i \\ v_2 = v_o \end{bmatrix} \quad i_2 = -G_L v_o$$

$-G_L v_2 =$ (i_2)

$$-G_L v_2 = y_{21} v_i + y_{22} v_2 \Rightarrow (-G_L - y_{22}) v_o = y_{21} v_i$$

$$\frac{v_o}{v_i} = \frac{-y_{21}}{G_L + y_{22}} = \frac{-y_{21}/G_L}{1 + y_{22}/G_L} = \frac{-N_{21}/D_{22}}{1 + N_{22}/D_{22}} = \frac{-N_{21}}{D_{22} + N_{22}}$$

Ex: $$\frac{v_o}{v_i}(s) = \frac{K}{s^6 + s^5 + 9s^4 + 6s^3 + 23s^2 + 8s + 15}$$ $\Rightarrow$ a low pass degree 6 filter



passive lossless

$$P_{ave}(j\omega) = 0 = \mathrm{Re}(V^* I)$$

$$= \frac{V^* I + V I^*}{2} = \frac{V^* y(j\omega) V + V y^*(j\omega) V^*}{2}$$

$$= |V|^2 \frac{\left(y(j\omega) + y^*(j\omega) \right)}{2} = 0$$

↑ lossless

$$y(+j\omega) = -y^*(j\omega) = -y(-j\omega)$$

real components ⇒ real coefficients in $y(s) = \frac{N(s)}{D(s)}$

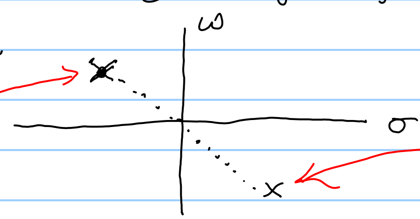
do an analytic continuation $\omega = s/j$

$$y(s) = -y(-s) \Rightarrow y(s) + y(-s) = 2\,Ev(y(s)) = 0\,;\ Ev = \text{Even}$$

⇒ $y(s)$ is odd

s plane $= \sigma + j\omega$

possible pole of $y(s)$



given possible pole of $y(-s)$ can't be as it is a pole of $y(s)$ which is assumed stable

$\therefore$ all poles of $y(s)$ (lossless) are on $j\omega$ axis have to be simple

$$y(s)\Big|_{\substack{\text{near a pole}\\ s=j\omega_0}} \approx \frac{k}{s-j\omega_0} \qquad \text{near } s=-j\omega_0 = s^* \quad = \frac{k^*}{s+j\omega_0}$$

$$y(s) \approx \frac{k}{s-j\omega_0} + \frac{k^*}{s+j\omega_0} + \text{other terms}$$

$$= \frac{s(k+k^*) + j\omega_0(k-k^*)}{s^2+\omega_0^2} + \ldots \Rightarrow k = k^*$$

$$= \frac{2ks}{s^2+\omega_0^2} + \ldots \text{ other similar terms } + \frac{k_0}{s} + k_\infty s$$

as $y(s)$ is odd then @ $s \to \infty$ $y(s) \approx k_\infty s$, $k_\infty > 0$
so a possible design is to remove poles at ∞ or of $1/y$ at ∞.

$$\frac{K}{s^6+s^5+9s^4+6s^3+23s^2+8s+15} = \frac{v_o}{v_i} = \frac{-y_{21}}{1+y_{22}} \quad \text{if } G_L = 1$$

$$= \frac{\overbrace{K/(s^5+6s^3+8s)}^{y_{21}}}{1+\underbrace{\dfrac{s^6+9s^4+23s^2+15}{s^5+6s^3+8s}}_{y_{22}}}$$

$$y_{22} = \frac{s^6+9s^4+23s^2+15}{s^5+6s^3+8s}$$

$\Rightarrow$ get a continued fraction expansion about $s=\infty$ (to give low pass

$$\begin{array}{r|l}
s^5+6s^3+8s & s^6+9s^4+23s^2+15 \quad \longrightarrow \; s \\
 & \underline{s^6+6s^4+8s^2} \\
 & 3s^4+15s^2+15
\end{array}$$

$$\begin{array}{r|l}
3s^4+15s^2+15 & s^5+6s^3+8s \quad \longrightarrow \; \tfrac{1}{3}s \\
 & \underline{s^5+5s^3+5s} \\
 & s^3+3s
\end{array}$$

$$\begin{array}{r|l}
s^3+3s & 3s^4+15s^2+15 \quad \longrightarrow \; 3s \\
 & \underline{3s^4 \quad\; 9s^2} \\
 & 6s^2+15
\end{array}$$

$$\begin{array}{r|l}
6s^2+15 & s^3+3s \quad \longrightarrow \; \tfrac{1}{6}s \\
 & \underline{s^3-\tfrac{5}{2}s} \\
 & \tfrac{1}{2}s
\end{array}$$

$$\frac{1}{2}s \,\overline{\smash{)}\, 6s^2 + 15} \quad 12s$$
$$6s^2$$
$$15 \,\overline{\smash{)}\, \tfrac{1}{2}s} \quad s/30$$

$$y_{22}(s) = s + \cfrac{1}{\frac{1}{3}s + \cfrac{1}{3s + \cfrac{1}{\frac{1}{6}s + \cfrac{1}{12s + \cfrac{1}{s/30}}}}}$$

$\leftarrow sy$

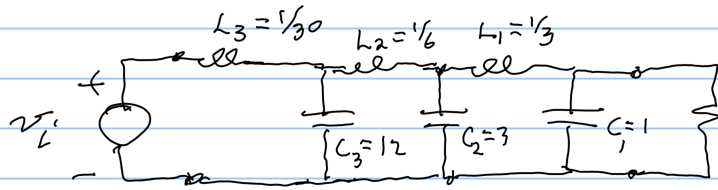
$\leftarrow sz$

$\leftarrow sy$

$\leftarrow sz$

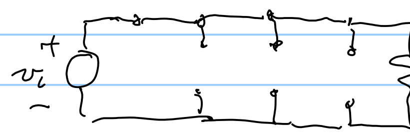
$\leftarrow sy$

$\leftarrow sz$



$+$ v_o $-$ $\Rightarrow \dfrac{v_o}{v_i} = \dfrac{K}{s^6 + \dots + 8s + 15}$

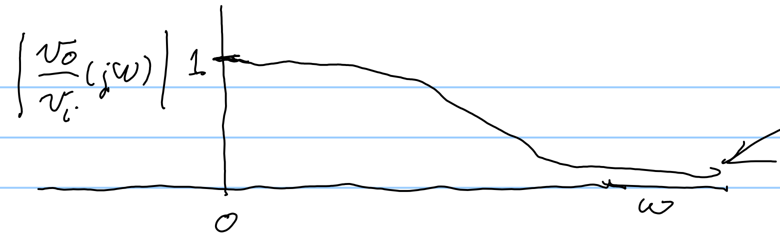
to get K, set $s = 0$



$+$ $v_o = v_i$ $-$ $\Rightarrow \left.\dfrac{v_o}{v_i}\right|_{s=0} = 1 = \dfrac{K}{15}$

$\Rightarrow K = 15$

gives $\dfrac{v_o}{v_i} = \dfrac{15}{s^6 + s^5 + 9s^4 + 6s^3 + 23s^2 + 8s + 15}$



drops to ∞ 6 times