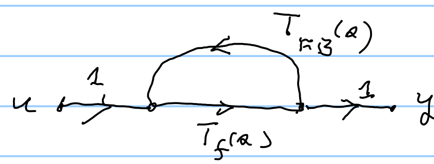
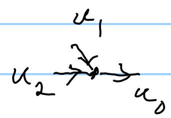
Note Title 10/17/2016

EE 303H

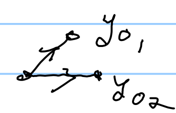
10/18/16

Feedback



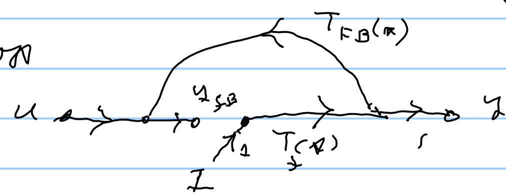


$u_0 = u_1 + u_2$



$y_{01} = y_{02}$

Break loop



$y_{FB}(\alpha)$ with unit input to $T_f(\alpha)$, $y_{FB}\big|_{u=0} = T_{FB} \cdot T_f$

return ratio is $y_{FB} = \dfrac{T_{FB} \cdot T_f \cdot 1}{1}$, return difference $\Gamma_{dif} = y_{FB} - 1$

for no input u, oscillations will be sustained if $y_{FB} = 1$

has at $s = j\omega_0$, $T_{FB}(j\omega)\, T_f(j\omega) = 1$ for $\omega = \omega_0$

$T_{FB} = 1$, $T_f(s) = N(s)/D(s)$, ratio of two polynomials

$$1 \cdot \frac{N(s)}{D(s)} = 1 \Rightarrow P(s) = N(s) - D(s) \Rightarrow -P(s) = D(s) - N(s)$$

if low pass, $N(s) = 1$, $D(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0$

Ex: $D(s) = s^3 + a_2 s^2 + a_1 s + a_0 - 1$; signals of the form $e^{j\omega t} = \cos\omega t + j\sin\omega t$

$$D(j\omega) = (a_0 - 1) + a_1 j\omega - a_2\omega^2 - j\omega^3$$

$$= \left((a_0 - 1) - a_2\omega^2\right) + j\left(a_1\omega - \omega^3\right)$$

$$= (a_0 - 1 - a_2\omega^2) + j\omega(a_1 - \omega^2)$$

desire $D(j\omega) \equiv 0$ at $\omega = \omega_0 \Rightarrow a_1 = \omega_0^2$ from imaginary part

from real part $\quad a_0 - 1 - a_2\omega_0^2 = 0 \Rightarrow a_0 - 1 - a_2 a_1 = 0$

$\Rightarrow a_0 = 1 + a_1 a_2 = 1 + \omega_0^2 a_2$ $\therefore$ can choose an a_2 real > 0

for stability

gives $\quad T_f(s) = \dfrac{1}{s^3 + a_2 s^2 + a_1 s + a_0} = \dfrac{1}{\underbrace{(a_1 s + s^3) + (a_0 + a_2 s^2)}_{\text{Hurwitz polynomial of stable}}}$

$$= \frac{1/(a_1 s + s^3)}{1 + \dfrac{(a_0 + a_2 s^2)}{a_1 s + s^3}} = \frac{1/N_f}{1 + D_f/N_f} = \frac{1/N_f}{1 + g(s)} = \frac{1}{s^3 + \cdots + a_0}$$

$$y(s) = \frac{a_0 + a_2 s^2}{a_1 s + s^3} = \frac{a_0 + a_2 s^2}{s(s^2 + a_1)} = \frac{k_1}{s} + \frac{k_2}{s + j\sqrt{a_1}} + \frac{k_2^*}{s - j\sqrt{a_1}}$$

$$k_1 = s\,y(s)\Big|_{s=0} \Rightarrow \frac{s(a_0 + a_2 s^2)}{s(s^2 + a_1)}\Big|_{s} = \frac{a_0}{a_1} = k_1 + \underbrace{\frac{s k_2}{s + j\sqrt{a_1}}\Big|_{s=0} + \frac{s k_2^*}{s - j\sqrt{a_1}}\Big|_{s=0}}_{\equiv 0}$$

$$k_1 = a_0/a_1 > 0$$

$$k_2 = (s + j\sqrt{a_1}) \cdot y(s)\Big|_{s=-j\sqrt{a_1}} = \frac{a_0 + a_2 s^2}{s(s + j\sqrt{a_1})(s - j\sqrt{a_1})} \times (s + j\sqrt{a_1})\Big|_{s=-j\sqrt{a_1}} = \frac{a_0 - a_2 a_1}{(-j\sqrt{a_1})(-2j\sqrt{a_1})}$$

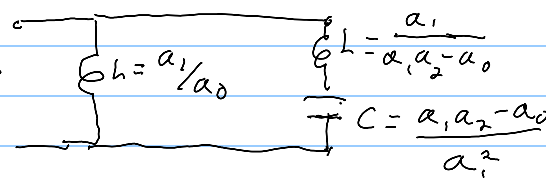
$$= \frac{a_0 - a_2 a_1}{-2a_1} = \frac{a_2}{2} - \frac{1}{2}\frac{a_0}{a_1} = \frac{1}{2}\left(\frac{a_1 a_2 - a_0}{a_1}\right)$$

$$y(s) = \frac{a_0 + a_2 s^2}{s(s^2 + a_1)} = \frac{a_0/a_1}{s} + \underbrace{\frac{\frac{1}{2}\left(\frac{a_1 a_2 - a_0}{a_1}\right)}{s + j\sqrt{a_1}} + \frac{\frac{1}{2}\left(\frac{a_1 a_2 - a_0}{a_1}\right)}{s - j\sqrt{a_1}}}_{\frac{a_1 a_2 - a_0}{a_1} \cdot s/(s^2 + a_1)}$$

$$y(s) = \frac{1}{\frac{a_1}{a_0} s} + \frac{\frac{(a_1 a_2 - a_0)}{a_1} s}{s^2 + a_1} = \frac{1}{\frac{a_1}{a_0} s} + \frac{1}{\left(\frac{a_1}{a_1 a_2 - a_0} s + \frac{a_1^2}{\frac{(a_1 a_2 - a_0) s}{a_1}} \right)}$$

a $Z(s)$

$y(s) \rightarrow$

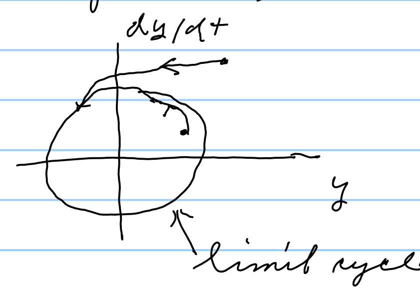


$$LC = \frac{s \cdot 1}{s} \cdot \frac{1}{a_1} = \frac{1}{a_1} = 1/\omega_0^2$$

$a_1 = \omega_0^2$

For a structurally stable oscillator look at the Van der Pol oscillator (at limit cycles of ODE's)

$$\frac{d^2 y}{dt^2} + \epsilon (y^2 - 1) \frac{dy}{dt} + \omega_0^2 y = 0$$



$$\frac{d}{dt}\left(\frac{dy}{dt} + F(\epsilon, y)\right) = -\omega_0^2 y \quad , \quad x = \frac{dy}{dt} + F(\epsilon, y)$$

$$\frac{dx}{dt} = -\omega^2 y$$

$$\frac{dy}{dt} = x - F(\epsilon, y)$$

$$\frac{dx}{dt} = -\omega_0^2 y \qquad \frac{dF(\epsilon,y)}{dt} = \epsilon(y^2-1)\frac{dy}{dt}$$

$$F(\epsilon, y) = \int_{y_0}^{y(t)} \epsilon(y^2-1)\,dy = \epsilon\frac{y^3}{3} - \epsilon y$$

$\therefore$

$$\left.\begin{aligned} \frac{dx}{dt} &= -\omega_0^2 y \\ \frac{dy}{dt} &= x - \epsilon\left(\frac{y^3}{3} - y\right) \end{aligned}\right\}$$ can make with Gilbert multipliers & capacitors

if on leftside: $C\frac{dv}{dt} = i \implies$ right side are currents depending on voltages

interested in the limit cycle: can make with OTA's