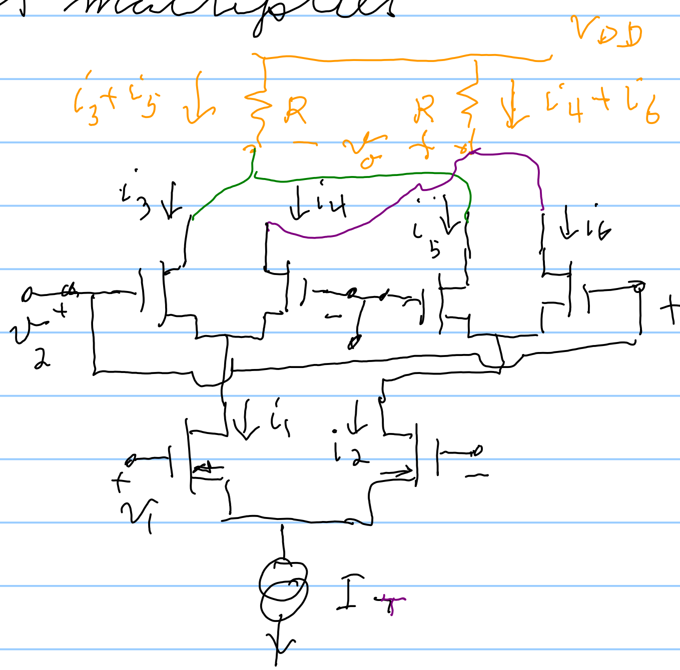
Note Title 10/13/2016

EE303H
10/13/16

Gilbert multiplier



$$0 = -v_o - R(i_4'+i_6') + R(i_3'+i_5')$$

$$v_o = R\left[(i_3'+i_5') - (i_4'+i_6')\right]$$

$$= R \cdot I_T\, f(v_1)\, f(v_2)$$

$i_1 = \frac{I_T}{2}(1 + f(v_1))$ $\quad$ $i_2 = \frac{I_T}{2}(1 - f(v_1))$

$i_3 = \frac{i_1}{2}(1 + f(v_2))$ $\quad$ $i_4 = \frac{i_1}{2}(1 - f(v_2))$

$$i_5 = \frac{i_2}{2}\left(1 - f(v_2)\right) \qquad i_6 = \frac{i_2}{2}\left(1 + f(v_2)\right)$$

$$i_3 + i_5 = \frac{I_T}{4}\left(1 + f(v_1)\right)\left(1 + f(v_2)\right) + \frac{I_T}{4}\left(1 - f(v_1)\right)\left(1 - f(v_2)\right)$$

$$= \frac{I_T}{4}\left[\left[1 + f(v_1) + f(v_2) + f(v_1)\cdot f(v_2)\right] + \left[1 - f(v_1) - f(v_2) + f(v_1)\cdot f(v_2)\right]\right]$$

$$= \frac{I_T}{2}\left[1 + f(v_1) f(v_2)\right]$$

$$i_4 + i_6 = \frac{I_T}{4}\left(\left[1 + f(v_1)\right]\left[1 - f(v_2)\right] + \left[1 - f(v_1)\right]\left[1 + f(v_2)\right]\right)$$

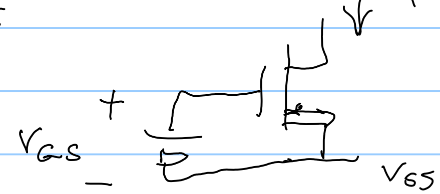
$$= \frac{I_T}{4}\left(1 + f(v_1) - f(v_2) - f(v_1)\cdot f(v_2) + 1 - f(v_1) + f(v_2) - f(v_1) f(v_2)\right)$$

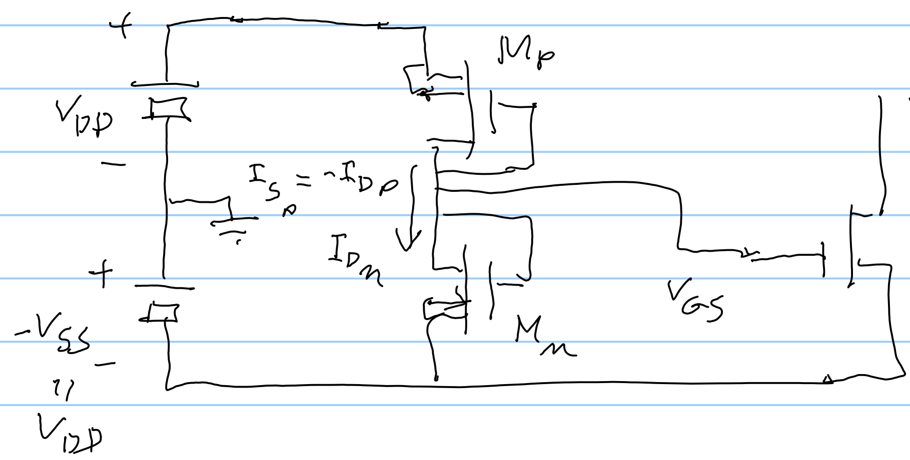
$$= \frac{I_T}{2}\left(1 - f(v_1)\cdot f(v_2)\right)$$

$$(i_3 + i_5) - (i_4 + i_6) = I_T\, f(v_1) f(v_2)$$ small signals $\approx k\, v_1 \cdot v_2$

Biasing for I_T

$$I_T \approx \frac{KP}{2} \cdot \frac{W}{L}\left(V_{GS} - VTO\right)^2$$





$$I_T = \frac{KP}{2}\frac{W}{L}\left(V_{GS} - VTO_n\right)^2$$

$$I_{D_p} = \frac{KP_p}{2}\left(\frac{W}{L}\right)_p \left((V_{DD} - V_{GS}) - |VTO_p|\right)^2$$

$$= \frac{KP_n}{2}\left(\frac{W}{L}\right)_n \left(V_{GS} - |VTO|_n\right)^2$$

$$\frac{(W/L)_p}{(W/L)_n} \times \frac{KP_p}{KP_n}\left(\frac{V_{DD} - V_{GS} - |VTO|_p}{V_{GS} - VTO_n}\right)^2 = 1$$

$$(W/L)_p = (W/L)_n \times \frac{KP_n}{KP_p}\left(\frac{V_{GS} - VTO_n}{V_{DD} - V_{GS} - |VTO_p|}\right)^2$$

gives a design formula to bias M_T

Differential equations

Linear time invariant

$$a_n \frac{d^n v_o}{dt^n} + a_{n-1} \frac{d^{n-1} v_o}{dt^{n-1}} + \dots + a_1 \frac{dv_o}{dt} + v_o = b_m \frac{d^m v_i}{dt^m} + \dots + b_1 \frac{dv_i}{dt} + b_0 v_i$$

if $v_i(t) = V_i e^{st}$ then a solution is $v_o(t) = V_o e^{st}$ for $-\infty < t < \infty$

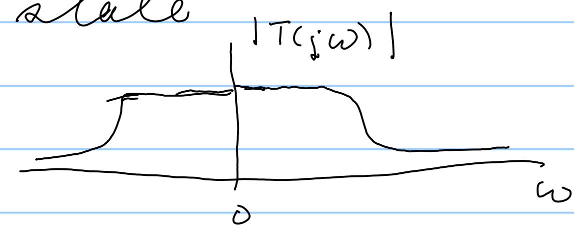
$$e^{st}\left[a_n s^n V_o + a_{n-1} s^{n-1} V_o + \dots a_1 s V_o + a_0 V_o\right] = \left[b_m s^m V_i + \dots + b_1 s V_i + b_0 V_i\right] e^{st}$$

$$\left[a_n s^n + a_{n-1} s^{n-1} + a_1 s + a_0\right] V_o = \left[b_m s^m + b_1 s + b_0\right] V_i$$

$$T(s) = \frac{V_o}{V_i}(s) = \frac{b_m s^m + \dots + b_1 s + b_0}{a_n s^n + \dots + a_1 s + a_0} \Rightarrow \text{the voltage transfer function}$$

set $s = j\omega \Leftrightarrow$ sinusoidal steady state

$$\frac{V_o}{V_i}(j\omega) = |T(j\omega)|\, e^{j\angle T(\omega)}$$



for low pass, $m < n$ normally $b_i = 0$ except $b_0 \neq 0$