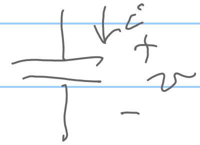
EE303H

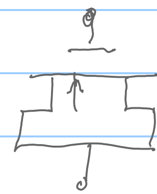
10/11/16

Note Title 10/11/2016

an MOS capacitor



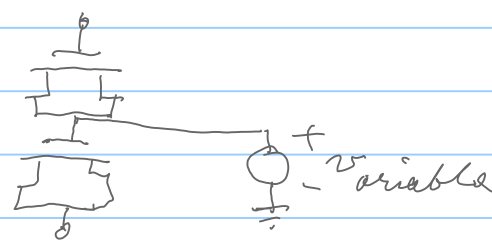
$i = C \dfrac{dv}{dt}$



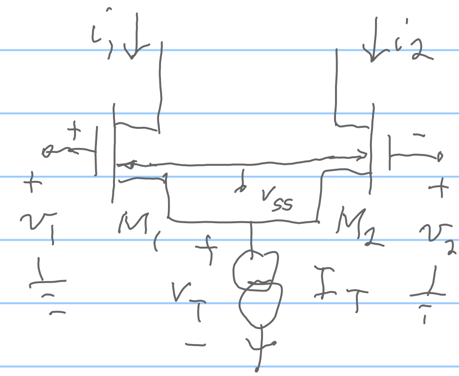
$C = W \times L \times C_{ox}$

$C_{ox} = \varepsilon_{si} \times \dfrac{1 \times 1}{t_{ox}}$ $W \times L = 1m \times 1m$ t_{ox} = thickness

v variable C



MOS differential pair



by KCL: $I_T = i_1 + i_2$

$\Rightarrow i_2 = I_T - i_1$

$v_1 - v_2 = v_{id}$

assume in saturation for M_1 & M_2, $M_1 = M_2$ $KP = \mu C_{ox}$

$i_1 = \frac{KP}{2} \cdot \frac{W}{L} (v_{GS_1} - V_{th})^2$ $i_2 = \frac{KP}{2} (v_{GS_2} - V_{th})^2$

$= \beta (v_1 - V_T - V_{th})^2$ $i_2 = \beta (v_2 - V_T - V_{th})^2$

$\sqrt{i_1/\beta} = v_1 - V_T - V_{th}$ $\sqrt{i_2/\beta} = v_2 - V_T - V_{th}$

$v_1 - v_2 = v_{id} = \sqrt{i_1/\beta} - \sqrt{i_2/\beta} \Rightarrow \sqrt{i_1} - \sqrt{i_2} = \sqrt{\beta}\, v_{id}$

square: $i_1 + i_2 - 2\sqrt{i_1 i_2} = \beta v_{id}^2 \Rightarrow 2\sqrt{i_1 i_2} = i_1 + i_2 - \beta v_{id}^2$

square again: $4 i_1 i_2 = i_1^2 + i_2^2 + 2 i_1 i_2 + (\beta v_{id}^2)^2 - 2\beta v_{id}^2 (i_1 + i_2)$

$$4(i_1)(I_T - i_1) = i_1^2 + (I_T - i_1)^2 + (\beta v_{id}^2)^2 - 2\beta v_{id}^2 I_T + 2 i_1 (I_T - i_1)$$

$\Rightarrow \quad 2 i_1 I_T - 2 i_1^2 = i_1^2 + I_T^2 + i_1^2 - 2 I_T i_1 + (\beta v_{id}^2)^2 - 2\beta v_{id}^2 I_T$

$\Rightarrow \quad 4 i_1^2 - 4 I_T i_1 + \underbrace{I_T^2 - 2 I_T (\beta v_{id}^2) + (\beta v_{id}^2)^2}_{(I_T - \beta v_{id}^2)^2} = 0$

$\Rightarrow$

$$i_1^2 - I_T i_1 + \frac{(I_T - \beta v_{id}^2)^2}{4} = 0$$

$\Rightarrow$ solving for i_1

$$i_1 = \frac{I_T}{2} \pm \frac{1}{2}\sqrt{\underbrace{(-I_T)^2 - 4(I_T - \beta v_{id}^2)^2/4}_{I_T^2 - I_T^2\left(1 - \frac{\beta}{I_T} v_{id}^2\right)^2}}$$

$$\underbrace{\qquad}_{+\left[2 \cdot 1 \times \frac{\beta}{I_T} v_{id}^2 - \left(\frac{\beta}{I_T} v_{id}^2\right)^2\right] I_T^2}$$

$\Rightarrow$

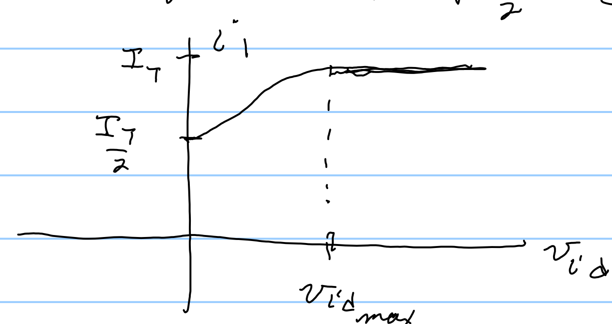
$$i_1 = \frac{I_T}{2}\left[1 \pm \sqrt{\frac{2\beta}{I_T} v_{id}^2 - \left(\frac{\beta}{I_T} v_d^2\right)^2}\right] = \frac{I_T}{2}\left[1 \pm \sqrt{\frac{2\beta}{I_T}}\, v_{id} \sqrt{1 - \frac{1}{2}\left(\frac{\beta}{I_T} v_{id}^2\right)}\right]$$

$i_{1\,max} = I_T \Rightarrow max\ v_{id}$ for this formula $i_1 \Rightarrow I_T$

here i_1 uses the + sign

$$i_1 = \frac{I_T}{2} + \sqrt{\frac{I_T}{2}\beta}\ v_{id}\sqrt{1 - \left(\sqrt{\frac{\beta}{2I_T}}\right)^2 (v_{id}^2)}$$

small signal $i_1 = \sqrt{\frac{I_T\beta}{2}}\, v_{id} \Rightarrow g_m = \sqrt{\frac{I_T\beta}{2}}$ for i_1 alone



for $v_{id_{max}}$ $\sqrt{\frac{I_T\beta}{2}}\ v_{id}\sqrt{1 - \left(\frac{\beta}{2I_T}\right)(v_{id})^2} = \frac{I_T}{2}$

if $\left(\frac{\beta}{2I_T}\right)\left(v_{id}^2\right)^2 = \frac{1}{2}$ gives $v_{id_{max}}$

$= v_{id}^2 = \frac{1}{2} \cdot \frac{2I_T}{\beta} = \frac{I_T}{\beta}$

$$i_2 = I_T - i_1 = \frac{I_T}{2} - \sqrt{\frac{I_T\beta}{2}}\, v_{id}\sqrt{1 - \left(v_{id} \times \sqrt{\beta/2I_T}\right)^2}$$

$$i_1 - i_2 = 2\sqrt{\frac{I_T\beta}{2}} \cdot v_{id}\sqrt{1 - \left(\sqrt{\beta/2I_T}\right)^2 v_{id}^2}$$

if $|v_{id}| \leq v_{id_{max}} \Rightarrow G_m = \sqrt{2I_T\beta}$

otherwise get $\pm I_T$