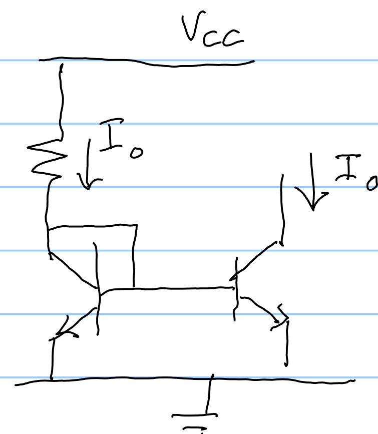
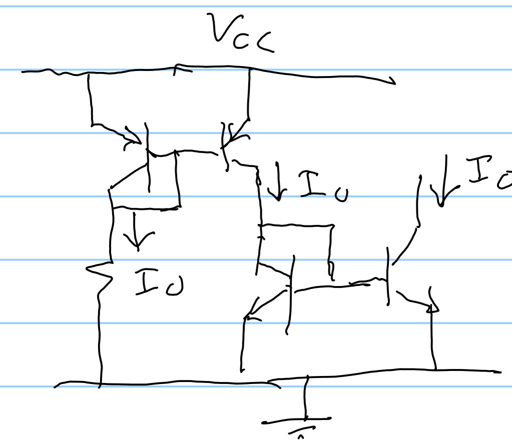
EE303H

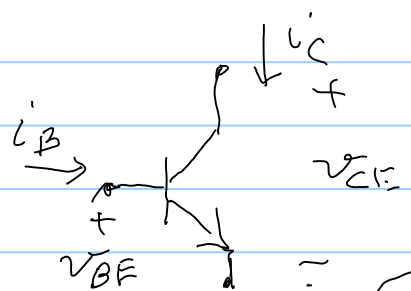
10/06/16

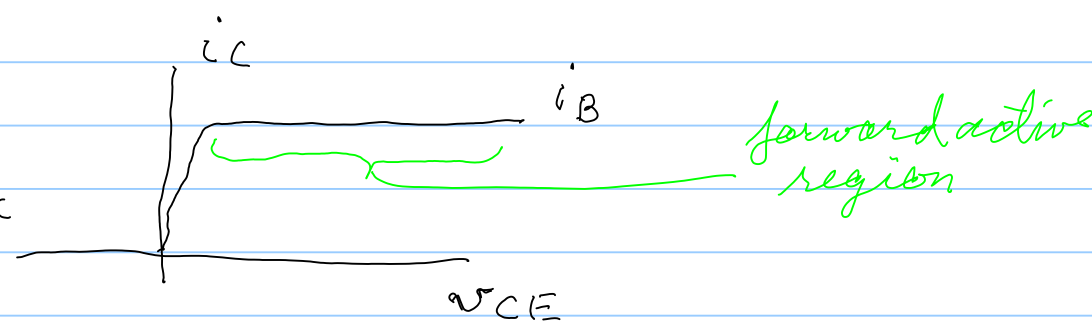
Note Title 10/6/2016

Current sink with BJT









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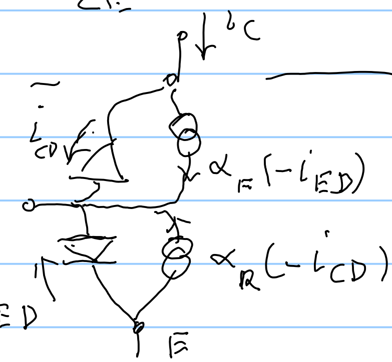
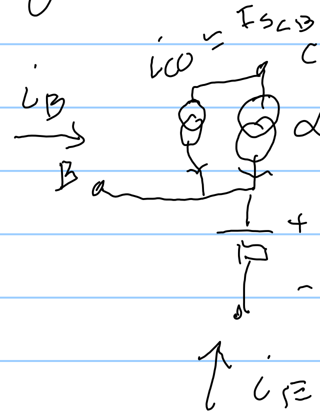


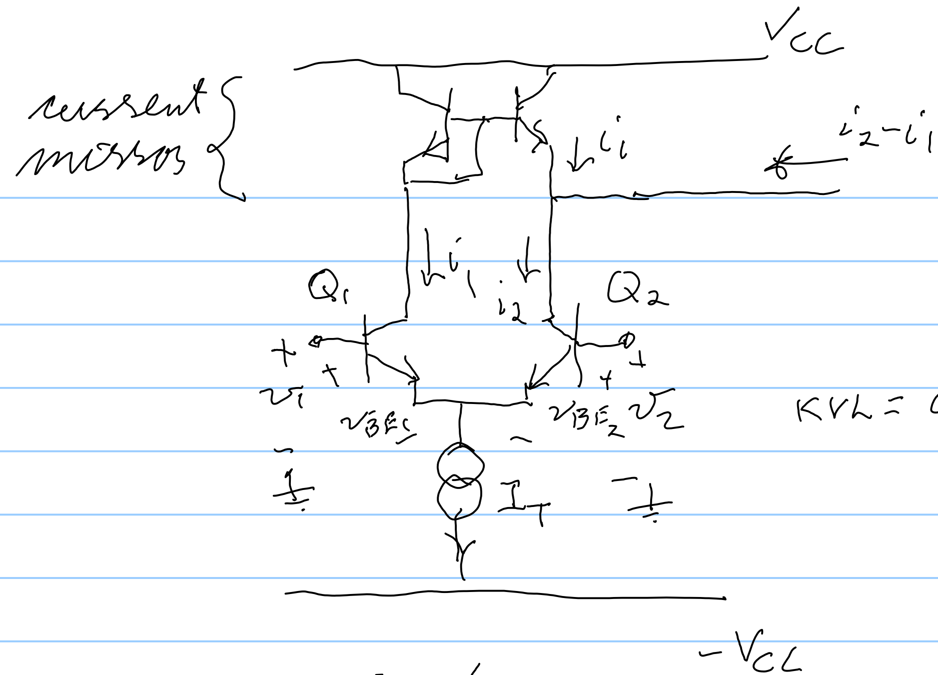
Ebers-Moll model

If we forward bias base to emitter & back bias base to collector



$$\alpha_F(-i_E) = \beta i_B = \boxed{\alpha_F I_{S_{BE}} e^{v_{BE}/V_T} = i_C} \Rightarrow \text{voltage controlled current}$$

$$v_{BE} \approx 0.7, \Rightarrow -i_E = I_{S_{BE}} e^{v_{BE}/V_T}$$



$Q_1 = Q_2$

$KVL = 0 = -v_1 + v_{BE_1} - v_{BE_2} + v_2 \Rightarrow v_1 - v_2 = v_{BE_1} - v_{BE_2}$

T = tail

$i_1 = \alpha I_S e^{v_{BE_1}/V_T}$; $i_2 = \alpha I_S e^{v_{BE_2}/V_T}$

V_T = thermal voltage $= kT/|q|$

$i_1 + i_2 = I_T$ $\qquad v_i = v_d = v_1 - v_2$

$\ln i_1 = (\ln \alpha I_S) + \ln(e^{v_{BE_1}/V_T})$; $\ln i_2 = \ln(\alpha I_S) + \ln(e^{v_{BE_2}/V_T})$

$\ln i_1 - \ln i_2 = \frac{v_{BE_1}}{V_T} - \frac{v_{BE_2}}{V_T} = v_i / V_T = \ln i_1/i_2$

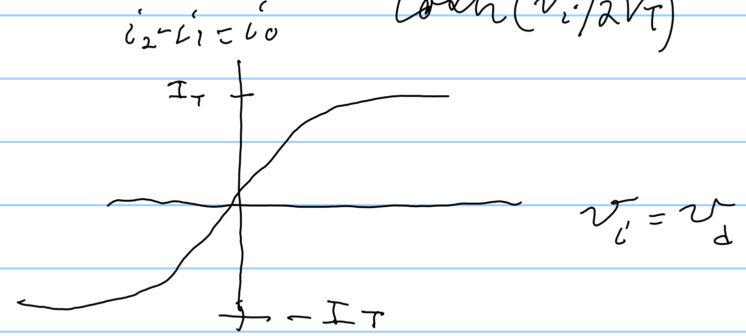
$$e^{\ln(i_1/i_2)} = \frac{i_1}{i_2} = e^{v_i/V_T}, \quad \text{but } i_1 = I_T - i_2$$

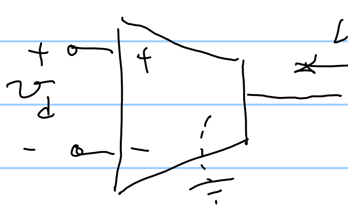
$$I_T - i_2 = e^{v_i/V_T} \cdot i_2 \Rightarrow i_2\left(1 + e^{v_i/V_T}\right) = I_T$$

$$i_2 = \frac{I_T}{1 + e^{v_i/V_T}}, \quad i_1 = I_T - i_2 = I_T\left(1 - \frac{1}{1 + e^{v_i/V_T}}\right) = \frac{I_T\, e^{v_i/V_T}}{1 + e^{v_i/V_T}}$$

$$i_2 - i_1 = I_T\left(\frac{1 - e^{v_i/V_T}}{1 + e^{v_i/V_T}}\right) = I_T \frac{e^{v_i/2V_T}}{e^{v_i/2V_T}}\left[\frac{\left(e^{-v_i/2V_T} - e^{+v_i/2V_T}\right)/2}{\left(e^{-v_i/2V_T} + e^{+v_i/2V_T}\right)/2}\right]$$

$$= I_T \frac{\sinh(v_i/2V_T)}{\cosh(v_i/2V_T)} = I_T \cdot \tanh(v_i/2V_T)$$

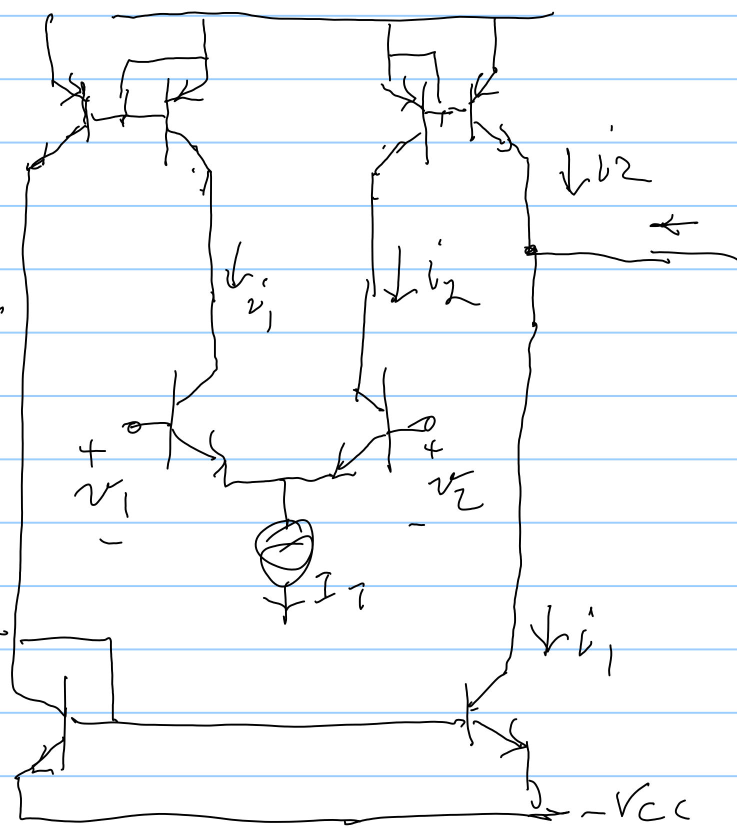


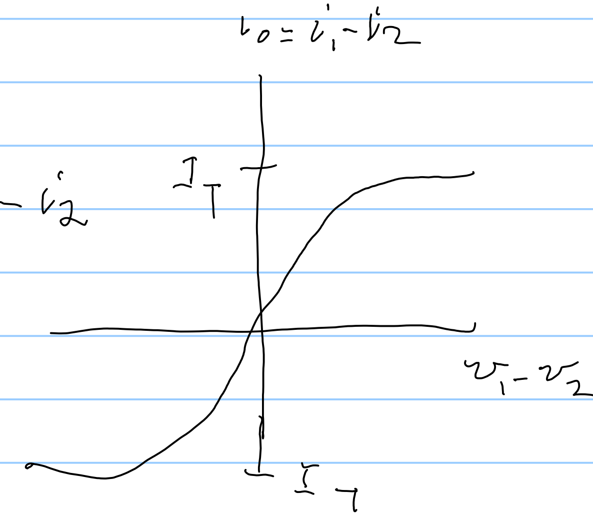


$$i_0 = I_T \tanh(v_d/2V_T)$$

called an OTA

operational transconductance amplifier





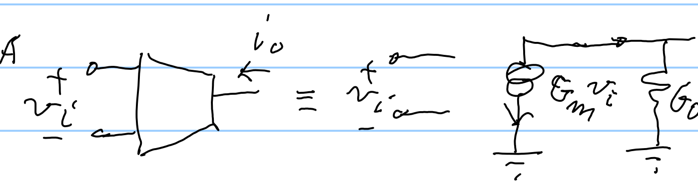
$i_O = I_T \tanh(v_i/2V_T)$

small signal $i_O = i_O(v_i=0) + \left.\frac{d I_T \tanh(v_i/2V_T)}{d v_i}\right|_{v_i=0} (v_i - 0) \leftarrow \ldots = I_O + G_m v_i + \ldots$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $\overset{\shortparallel}{0}$

$$G_m = \left.\frac{d\, I_T \tanh(v_i/2V_T)}{d v_i}\right|_{v_i=0} = I_T \cdot \frac{1}{2V_T} \left.\frac{d(\tanh x)}{dx}\right|_{x=0,\ x=v_i/2V_T} = \frac{I_T}{2V_T}\left(1-\tanh^2 x\right)$$

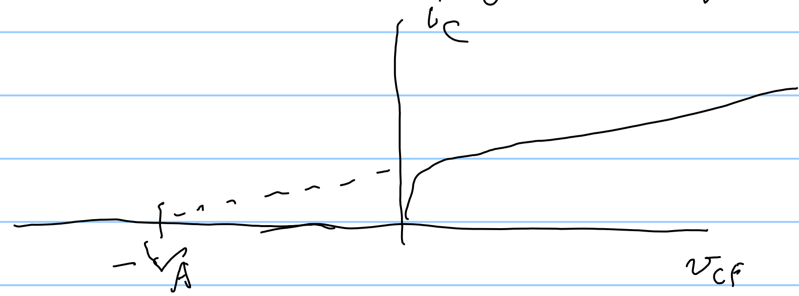
$$= \frac{I_T}{2V_T}$$

small signal OTA



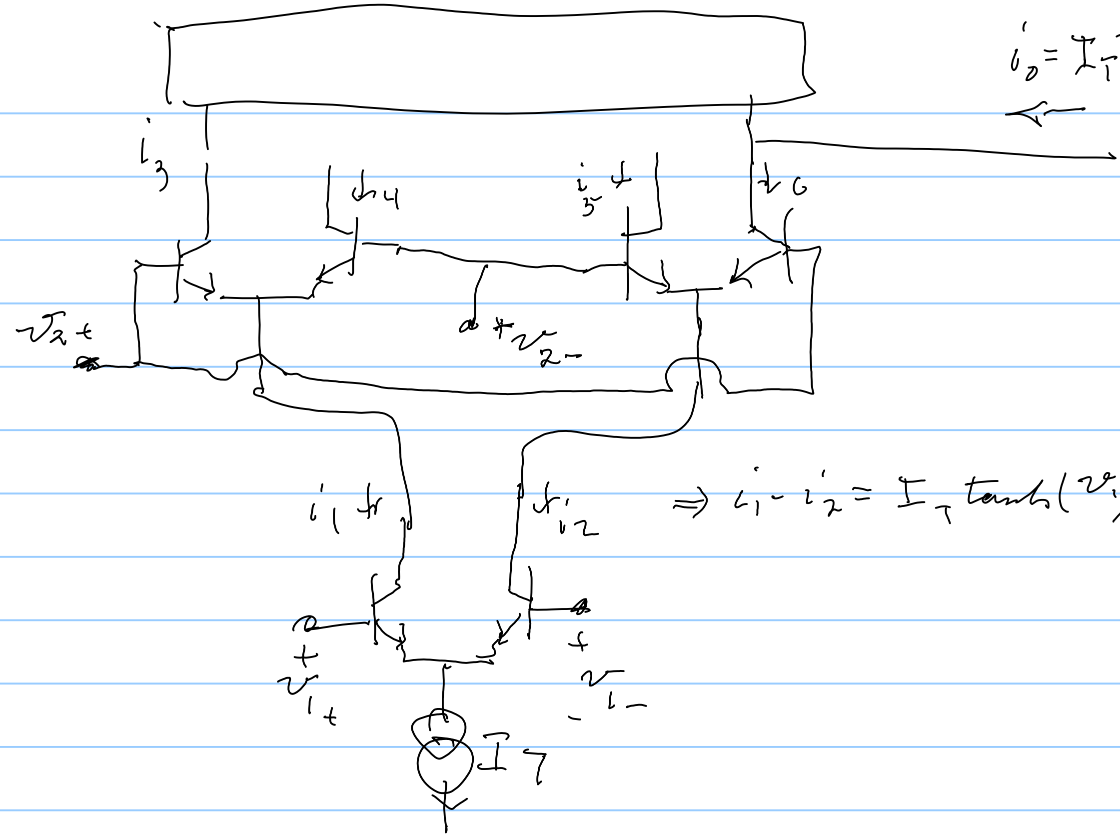
$$G_0 = \frac{I_T}{V_A}$$

V_A = Early voltage



For a 4-quadrant multiplier use I_{T_1} & $\bar{I}_{T_2}$ as inputs to to 2-differential & get I_{T_1} & $\bar{I}_{T_2}$ as currents i_1 & i_2 of one differential pair

$$i_o = I_T \tanh\frac{v_1}{2V_T} \cdot \tanh\frac{v_2}{2V_T}$$



$$\Rightarrow i_1 - i_2 = I_T \tanh(v_1/2V_T)$$