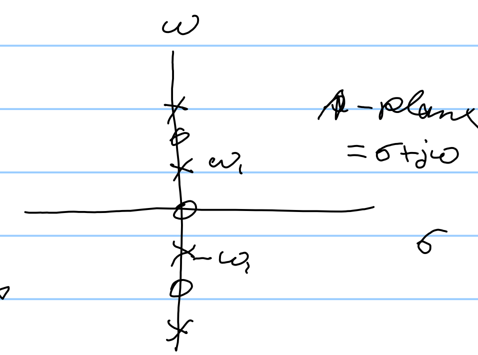
Note Title 12/10/2015

EE 610
12/10/15

PR functions, lossless
2-port Y's, Z's
Indefinite Y
graphs for circuits

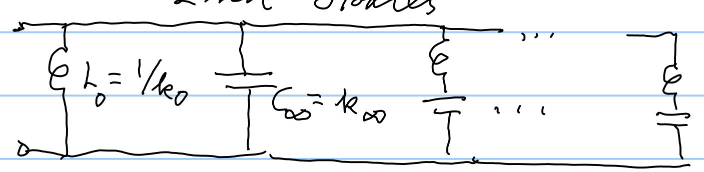



partial fraction expansion

$$PR \Rightarrow \text{lossless} = y(s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{N} \frac{2k_i s}{s^2+\omega_i^2}$$

$k_i$ real & positive
$i = 0, \infty, 1, \dots, N$

2nd Foster

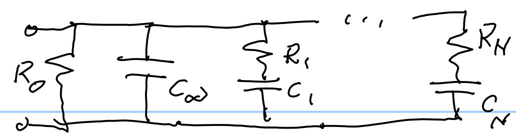



$$L_i = \frac{1}{2k_i}, \quad C_i = \frac{2k_i}{\omega_i^2}$$

$$L_i \cdot C_i = 1/\omega_i^2$$

$$y_i(s) = \frac{2k_i s}{s^2+\omega_i^2} = \frac{1}{\frac{s}{2k_i} + \frac{\omega_i^2}{2k_i s}} = \frac{1}{z_i(s)}$$

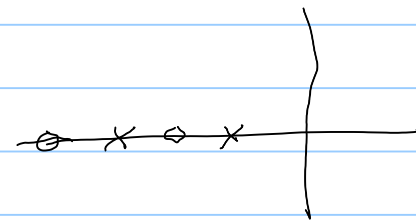
to get an RC 2nd Foster replace every L by an R



$$= Y_{RC} = G_0 + k_\infty s + \sum Y_{i_{RC}}$$

$$Y_{i_{RC}} = \frac{1}{L_i s\big|_{s=1} + \frac{1}{C_i s}} = \frac{1}{R_i + \frac{1}{C_i s}} = \frac{C_i s}{R_i C_i s + 1} = \frac{G_i s}{s + 1/R_i C_i}$$

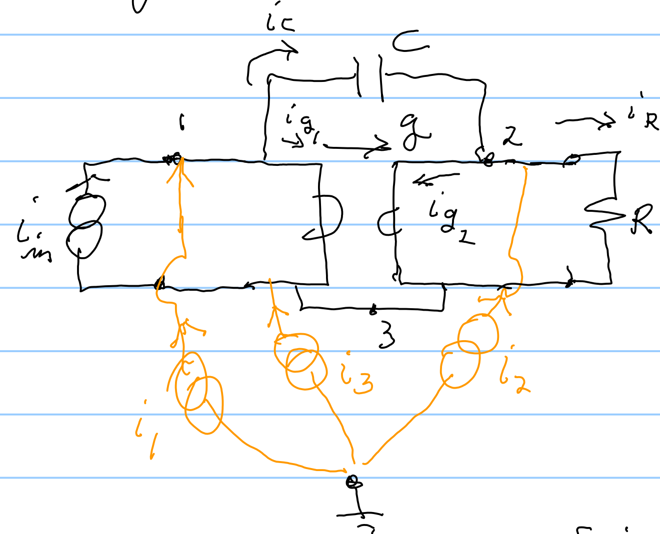
pole @ $-1/R_i C_i$



$$Y_{RC} = G_0 + C_\infty s + \sum_{i=1}^{N} \frac{G_i s}{s + 1/R_i C_i} \Rightarrow \frac{Y_{RC}}{s} = \frac{G_0}{s} + C_\infty + \sum \frac{G_i}{s + 1/RC}$$

is a partial fraction expansion

Indefinite admittance: $\Sigma$ of row elements $= 0 = \Sigma$ of column elements




KCL:

$$i_{in} + i_1 = i_C + i_{g_1} = sC(v_1 - v_2) + (-g)(v_2 - v_3)$$

$$i_2 = i_{g_2} + i_R + (-i_C) = g(v_1 - v_3) + G(v_2 - v_3) + sC(v_2 - v_1)$$

$$i_3 =$$

$$\begin{bmatrix} i_{in} + i_1 \\ i_2 \\ -i_{in} + i_3 \end{bmatrix} = \underbrace{\begin{bmatrix} sC & -sC - g & g \\ -sC + g & sC + G & -g - G \\ -g & g - G & G \end{bmatrix}}_{Y_{ind}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

if ground is @ node 3, $v_3 = 0$ & can ignore 3rd column also ignore $i_3$ as it goes into ground

$$Y_{def} = \begin{bmatrix} sC & -sC-g \\ -sC+g & sC+G \end{bmatrix} \Rightarrow \begin{bmatrix} i_{in}+i_1 \\ i_2 \end{bmatrix} = Y_{def} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

to get $y_{in}$ seen by $i_{in}$ set $i_1 = i_2 = 0$

$$\begin{bmatrix} i_{in} \\ 0 \end{bmatrix} = \begin{bmatrix} \overset{Y_{11}}{sC} & \overset{Y_{12}}{-sC-g} \\ \underset{Y_{21}}{-sC+g} & \underset{Y_{22}}{sC+G} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \text{2nd row}$$

$$-(sC+G)v_2 = (-sC+g)v_1$$

$$\Rightarrow v_2 = \frac{-(-sC+g)}{sC+G} \cdot v_1 \Rightarrow v_2 = -Y_{22}^{-1} \cdot Y_{21} \Rightarrow Y_{11} - Y_{12} \cdot Y_{22}^{-1} Y_{21}$$

$$i_{in} = sCv_1 + \frac{(-sC-g) \cdot (-1)(-sC+g)}{sC+G} \cdot v_1$$

$$y_{in} = \frac{sC + (sC+g)(-sC+g)}{sC+G} = \frac{\cancel{(sC)^2} + (sCG) - \cancel{(sC)^2} + g^2 + \cancel{sCg} - \cancel{sCg}}{sC+G}$$

$$= \frac{sCG+g^2}{sC+G} = \frac{s + g^2/CG}{s + G/C} \times G \qquad \text{if } G=0 \Rightarrow R=\infty;\ y_{in} = g^2/sC = \frac{1}{sL_{eq.}}$$

$L_{eq} = C/g^2$

$-i_{in} + i_3 = -g v_1 + (g-G) v_2 + G \times 0$

$= -g \cdot \frac{1}{y_{in}} \cdot i_{in} + (g-G) \cdot \left[\frac{sC-g}{sC+G}\right] \cdot \frac{1}{y_{in}} i_{in} \Rightarrow i_3 = \left[1 - \frac{g}{y_{in}} + \frac{(g-G)}{y_{in}}\left(\frac{sC-g}{sC+G}\right)\right] i_{in}$

graphs: laws of connections go with the graphs

KCL & KVL $\quad P_{in} = 0 = v_b^T \cdot i_b$ if inclose the circuit

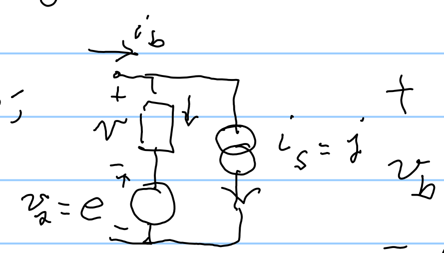
" $\quad$ " $\sum v$'s around a closed path $= 0$

$\sum i$'s into a closed sphere $= 0$

trees give minimum number of KCL's $\Rightarrow$ $0_t = C i_b$ $\quad v_b = C^T v_t$

cotrees " " " of KVL's $\quad 0_\ell = \mathcal{J} v_b$ $\quad i_b = \mathcal{J}^T i_\ell$

also need laws of elements;




$v = v_b - e, \quad i = i_b - j \quad \Rightarrow \quad Av = Bi$ if $A^{-1}$ exists then $Z_{b \times b} = A^{-1} B$

can get state eqs. $\Rightarrow$ semistate $\quad E\dot{x} = A(x) + Bu$

$\quad y = Cx$