Note Title

EE610
10/22/15

10/22/2015

$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

circulator
3-port

$Av = Bi \;;\quad Yv = i$

$(B+A)^{-1}(B-A) = S = (1_3 + Y)^{-1}(1_3 - Y)$

$(1_3 + Y)S = S + YS = 1_3 - Y$

$S - 1_3 = -YS - Y = -Y(S + 1_3)$

$(1_3 - S)(1_3 + S)^{-1} = Y$

$$(1_3 + S)^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\det = 1 \cdot \det\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - 1 \det\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

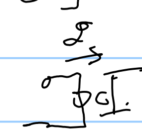
$$= 1 \cdot 1 - 1(-1) = 2$$

$$(1_3 - S) = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$Y = (1_3 + S)^{-1}(1_3 - S) = \frac{1}{2}\begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & 2 \\ 2 & -2 & 0 \end{bmatrix}$$

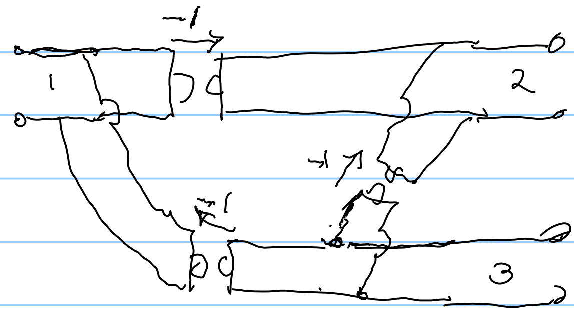
$$= \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$y_{12} = 1, \; y_{21} = -1$

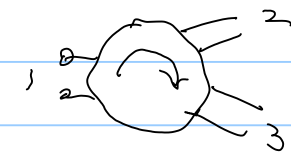


$$Y = \begin{bmatrix} 0 & -g \\ +g & 0 \end{bmatrix} \quad g = -1$$

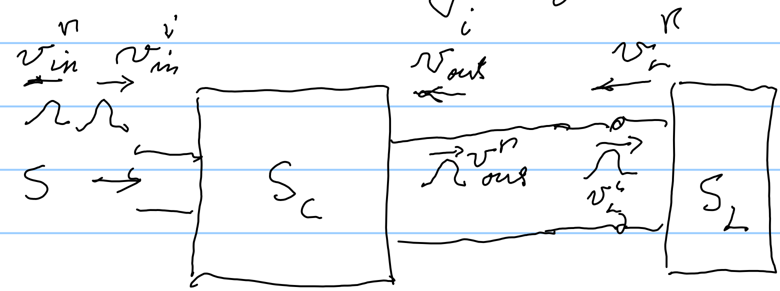
Can make with 3 gyrators



$=$



Loaded scattering system



$$S_c = \left[\begin{array}{c|c} S_{11} & S_{12} \\ \hline S_{21} & S_{22} \end{array}\right] = \left[\begin{array}{c|cc} 0 & 0 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]$$

$$S_L = \begin{bmatrix} S_a & 0 \\ 0 & S_b \end{bmatrix} \Rightarrow S = S_b \cdot S_a$$

$v_L^{r} = v_{out}^{i}$ when normalize

$v_L^{i} = v_{out}^{r}$ to the same Z_0

$$S_C \begin{bmatrix} v_{in}^{i} \\ v_{out}^{i} \end{bmatrix} = \begin{bmatrix} v_{in}^{r} \\ v_{out}^{r} \end{bmatrix} \qquad v_{in}^{r} = S\, v_{in}^{i} = S_{11} v_{in}^{i} + S_{12} v_{out}^{i}$$

$$v_{out}^{r} = S_{21} v_{in}^{i} + S_{22} v_{out}^{i}$$

$$v_L^{r} = v_{out}^{i} = S_L \,, \quad v_L^{i} = S_L v_{out}^{r}$$

$$S_L v_{out}^{r} = v_{out}^{i} = S_L S_{21} v_{in}^{i} + S_L S_{22} v_{out}^{i}$$

$$\Rightarrow \left(1_L - S_L S_{22}\right) v_{out}^{i} = S_L S_{21} v_{in}^{i}$$

$$v_{out}^{i} = \left(1_L - S_L S_{22}\right)^{-1} S_L S_{21} v_{in}^{i}$$

$$S\, v_{in}^{i} = \left[S_{11} + S_{12} \left(1_L - S_L S_{22}\right)^{-1} S_L S_{21} \right] v_{in}^{i}$$

$$S = S_{11} + S_{12} \left(1_L - S_L S_{22}\right)^{-1} S_L S_{21}$$

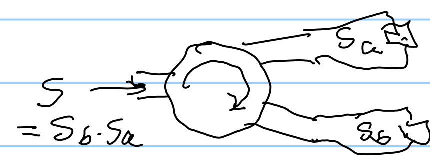
Ex

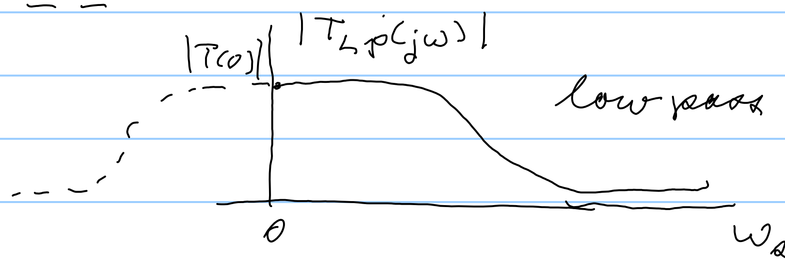
$$1_L - S_L S_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} S_a & 0 \\ 0 & S_b \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -S_b & 1 \end{bmatrix} \Rightarrow \left(1_L - S_L S_{22}\right)^{-1} = \begin{bmatrix} 1 & 0 \\ S_b & 1 \end{bmatrix}$$

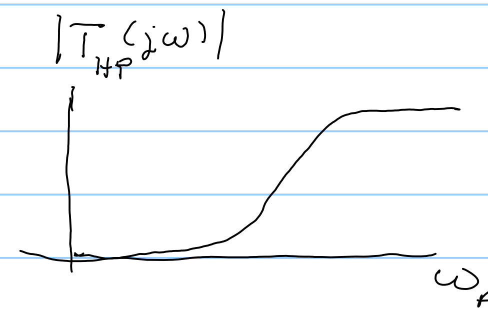
$$S_L S_{21} = \begin{bmatrix} S_a & 0 \\ 0 & S_b \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} S_a \\ 0 \end{bmatrix},$$

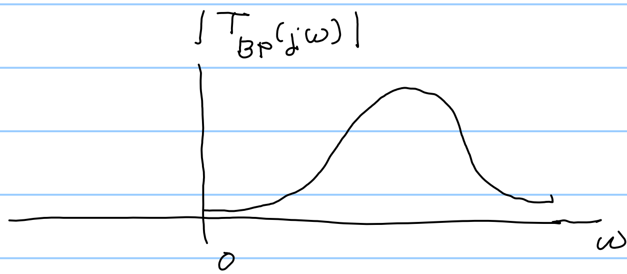
$$S = S_{11} + S_{12}(1_L - S_L S_{22})^{-1} S_L S_{21} = [0] + [0 \quad 1] \begin{bmatrix} 1 & 0 \\ S_b & 1 \end{bmatrix} \begin{bmatrix} S_a \\ 0 \end{bmatrix} = [S_b \quad 1] \begin{bmatrix} S_a \\ 0 \end{bmatrix} = S_b \cdot S_a$$

$S = S_b \cdot S_a$







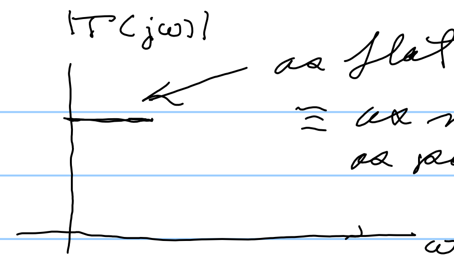


$\Rightarrow$ use $\alpha = 1/p$

(preserves PR property)

$$p/\omega_0 + \frac{\omega_0}{p} = \alpha$$

Maximally flat LP filter



as flat as possible ≡ as many derivatives as possible (in ω) are 0

$$T(s) = \frac{K}{s^n + d_{n-1} s^{n-1} + \dots d_1 s + d_0}$$

$$T(j\omega) \cdot T^*(j\omega) = |T(j\omega)|^2 = T(j\omega)\, T(-j\omega)$$

as real coefficients so all * are on s

$$\frac{d\,|T(j\omega)|^2}{d\omega} = 2|T(j\omega)| \cdot \frac{d|T(j\omega)|}{d\omega}$$

so for $\frac{d|T(j\omega)|}{d\omega} = 0$ can use $\frac{d|T(j\omega)|^2}{d\omega} = 0$

$$|T(j\omega)| = \frac{|K|}{|D(j\omega)|} \; ; \quad \frac{d|T(j\omega)|}{d\omega} = \frac{-|K|}{|D(j\omega)|^2} \frac{d|D(j\omega)|}{d\omega}$$

$$\therefore \frac{d|T(j\omega)|}{d\omega} = 0 \Rightarrow \frac{d|D(j\omega)|}{d\omega} = 0 \Rightarrow \frac{d|D(j\omega)|^2}{d\omega} = 0$$

also know that $|D(j\omega)| \neq 0$ for $-\infty < \omega < \infty$

$$D(s) = s^n + d_{n-1} s^{n-1} + \dots + d_1 s + d_0$$

$$D(j\omega) = j^n \omega^n + j^{n-1} d_{n-1} \omega^{n-1} + \dots + j d_1 \omega + d_0$$

$$D(j\omega)\cdot D(-j\omega) = d_0^2 + \alpha_1 \omega^2 + \alpha_2 \omega^4 + \dots + \alpha_{2n-1} \omega^{2n}$$

$$= d_0^2 + \underbrace{\frac{d|D(j\omega)|^2}{d\omega^2}}_{=0}\omega^2 + \dots + \underbrace{\frac{d^{2n}|D(j\omega)|^2}{d\omega^{2n-1}}}_{=0} + \alpha_{2n}\omega^{2n}$$

as max flat

$$= d_0^2 + \alpha_{2n}\omega^{2n} = d_0^2 + (j)^{2n}\omega^{2n} = d_0^2 + (-1)^n \cdot \omega^{2n}$$

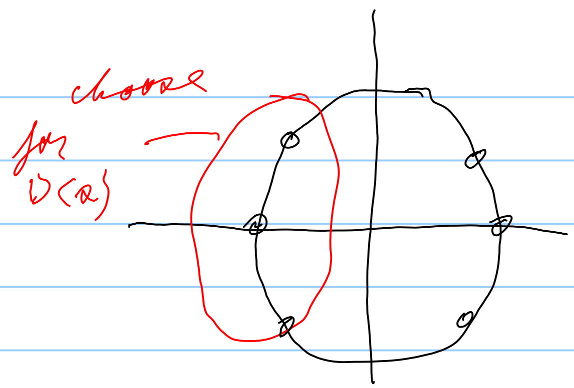
$\omega = \alpha/j$ $$|D(j\omega)|^2\Big|_{\omega=\alpha/j} = d_0^2 + (-1)^n\left(\frac{\alpha}{j}\right)^{2n} = d_0^2 + \alpha^{2n}$$

normalize for $d_0 = 1$ $\quad P(\alpha) = \alpha^{2n} + 1 = D(\alpha)D(-\alpha)$

desire to factor $P(\alpha)$ into $D(\alpha)D(-\alpha) \Rightarrow$ desire the zeros of $P(\alpha)$ they are $2n$ th roots of $-1 \Rightarrow P(\alpha) = 0 \Rightarrow \alpha^{2n} = -1$

$$-1 = e^{j\pi} = e^{j(\pi + 2k\pi)} = \alpha^{2n} \qquad k = 0, \pm 1, \dots$$

m th root is $A_m = e^{j(\pi + 2m\pi)/2n}$



α-plane all roots are on the unit circle

for $D(\alpha)$, as desire $T(\alpha) = \frac{K}{D(\alpha)}$ stable

we want left half plane roots