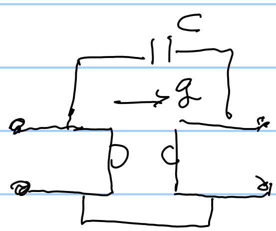
Note Title 10/20/2015

EE 610
10/20/15



$$y_\ell = \frac{g^2 - aC\,y(a)}{y(a) - aC} \qquad R_y = \frac{k\,y(k) - a\,y(a)}{k\,y(a) - a\,y(k)} \times y(k)$$

$$= g^2 \left(\frac{1 - \frac{aC}{g}\cdot\frac{y(a)}{g}}{y(a) - aC} \right) \qquad = \frac{k\,y^2(k)\left(1 - \frac{a}{k}\cdot\frac{y(a)}{y(k)}\right)}{k\,y(k)\left(\frac{y(a)}{y(k)} - a/k\right)} = y(k)\left(\frac{1 - \frac{a}{k}\cdot\frac{y(a)}{y(k)}}{\frac{y(a)}{y(k)} - \frac{a}{k}}\right),$$

$$= g \left(\frac{1 - \frac{aC}{g}\cdot\frac{y(a)}{g}}{\frac{y(a)}{g} - \frac{aC}{g}} \right)$$

set 等 $\Rightarrow y_\ell = R_y$

to identify set $g = y(k)$, $\frac{C}{g} = \frac{1}{k}$; k = zero of $\mathrm{Ev}\,y(a)$ assume given so know $y(k)$

$$C = g/k = y(k)/k$$

$\therefore$ choose k = zero of $\mathrm{Ev}\, y(s)$ with k real

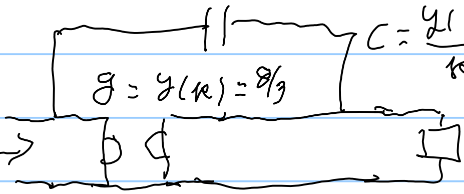
If $y(s)$ is lossless PR then $y(s)$ is odd $\Rightarrow$ every s is a zero of $\mathrm{Even}\, y(s)$. $\Rightarrow$ can use any $k > 0$

allows synthesis of any lossless PR $y(s)$

Ex: $y(s) = \dfrac{s(s^2+4)}{(s^2+2)}$; choose $k = 2$, $y(2) = \dfrac{2(4+4)}{(4+2)} = \dfrac{16}{6} = \dfrac{8}{3}$

$$y(s) + y(-s) = 2\,\mathrm{Ev}\, y(s) = \frac{s(s^2+4)}{(s^2+2)} + \frac{(-s)((-s)^2+4)}{((-s)^2+2)} = 0$$

Form

$y = \dfrac{s(s^2+4)}{s^2+2}$ $\rightsquigarrow$ 

$C = \dfrac{y(k)}{k} = \dfrac{8/3}{2} = 4/3$

$y_e = 4\left(\dfrac{s^2+8}{s}\right)$

$$R_y = \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)} \times y(k) = y_e(s) = \frac{2 \times \frac{8}{3} - s\left(s\left(\frac{s^2+4}{s^2+2}\right)\right)}{2 \times \frac{s(s^2+4)}{s^2+2} - \frac{8}{3}s} \times 8/3$$

$$y_\ell(\lambda) = \frac{8}{3}\left[\frac{16(\lambda^2+2) - 3\lambda^2(\lambda^2+4)}{6\lambda(\lambda^2+4) - 8\lambda(\lambda^2+2)}\right] = \left(\frac{\lambda^2-4}{\lambda^2-4}\right)\cdot y_x$$

$$= \frac{8}{3}\left[\frac{-3\lambda^4 + (16-12)\lambda^2 + 32}{(6-8)\lambda^3 + (24-16\lambda)}\right] = \frac{8}{3}\left[\frac{3\lambda^4 - 4\lambda^2 - 32}{2\lambda^3 - 8\lambda}\right]$$

$$= \frac{8}{3}\left[\frac{(3\lambda^2+8)(\lambda^2-4)}{2\lambda(\lambda^2-4)}\right] = \frac{8}{3}\left(\frac{3\lambda^2+8}{2\lambda}\right) = 4\left(\frac{\lambda^2 + 8/3}{\lambda}\right)$$

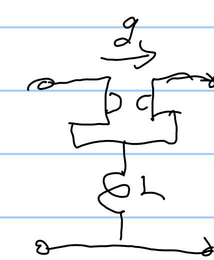
lossless

$\delta[y_\ell(\lambda)] = 2$

$\delta[\cdot] =$ degree $\quad \delta[y'(\lambda)] = 3$

$$\begin{array}{r|l} & 3\lambda^2 + 8 \\ \lambda^2 - 4 & 3\lambda^4 - 4\lambda^2 - 32 \\ & 3\lambda^4 - 12\lambda^2 \\ \hline & 8\lambda^2 - 32 \\ & 8\lambda^2 - 32 \end{array}$$

Dual for another type of Richards' function



Reason this works $\Rightarrow$ $R_y(s)$ is PR and $y(s)+y(-s)=0$ @ $s=k>0$

as then s^2-k^2 cancels in $R_y(s)$

& $y_e = R_y(s)$ has $\delta[y_e]=\delta[y]-1$

Look at the scattering matrix = reflection coefficient that goes with $R_y(s)$

$$Av = Bi \Rightarrow y = B^{-1}A, \quad S = (B+A)^{-1}(B-A)$$

$$= \left(B\left[1_n + B^{-1}A\right]\right)^{-1} \cdot B\left[1_n - B^{-1}A\right]$$

$$= \left[1_n + B^{-1}A\right]^{-1} B^{-1} \cdot B\left[1_n - B^{-1}A\right]$$

$$= \left(1_n + y\right)^{-1}\left(1_n - y\right)$$

$$S = \left(1 + R_y/y(k)\right)^{-1}\left(1 - R_y/y(k)\right)$$

$$= \left(1 + \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}\right)^{-1}\left(1 - \frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}\right)$$

$$= \left(k\,y(s) - s\,y(k) + k\,y(k) - s\,y(s)\right)^{-1}\left(k\,y(s) - s\,y(k) - k\,y(k) + s\,y(s)\right)$$

$$= \Big((k-s)\left[y(s) + y(k) \right] \Big)^{-1} \Big((k+s)\left[y(s) - y(k) \right] \Big)$$

$$S(s) = \left(\frac{k+s}{k-s}\right)\left[\frac{y(s)-y(k)}{y(s)+y(k)}\right] = \left(\frac{s+k}{s-k}\right)\underbrace{\left[\frac{1 - y(s)/y(k)}{1 + y(s)/y(k)}\right]}_{S \text{ for } y(s) \text{ is } S \text{ is BR}}$$

the pole @ $s = k$ cancels

numerator & denominator $\Rightarrow S_{R_y}$ is still BR

$$\left| S_{R_{y/y(k)}}(j\omega) \right| = \left| \frac{j\omega + k}{j\omega - k} \right| \left| S_y(j\omega) \right| = \frac{\sqrt{\omega^2 + k^2}}{\sqrt{\omega^2 + k^2}} \cdot \left| S_y(j\omega) \right| = \left| S_y(j\omega) \right|$$

$$\Rightarrow S_{R_y} \text{ is BR if } y \text{ is PR} \Rightarrow y_r = y(k) R_y(s) \text{ is PR}$$

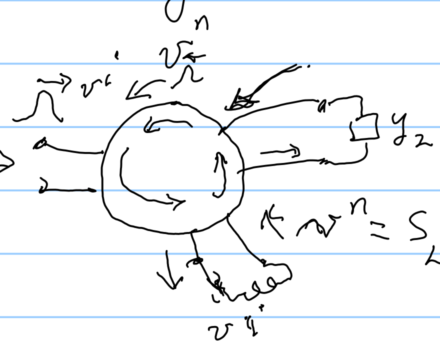
also if k is a zero of $Ev\, y(s)$ then another factor cancels

this shows up in $S_R = \underbrace{\left(\frac{s-k}{s+k}\right)}_{\text{is bounded real}} \underbrace{S_2}_{\text{BR}}$; $\delta[S_2(s)] = \delta[S(s)] - 1$

$$= \frac{s/k - 1}{s/k + 1} \cdot S_2 = \left(\frac{sh - 1}{sh + 1}\right) \cdot \left(\frac{1 - y_2}{1 + y_2}\right)$$

The product of two S's is realized via a circulator

$S = S_L \cdot S_{y_2} \rightarrow$



$v^n = S_L \cdot v^i = v^i_{y_2}$; $v^n_{y_2} = S_{y_2} \cdot v^i_{y_2} = \underbrace{S_{y_2} \cdot S_L}_{S} v^i = v^n$

$S v^i = v^n$

$$S_{\text{circulator}} \begin{bmatrix} v^i \\ v^i_{y_L} \\ v^i_{y_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v^i \\ v^i_{y_L} \\ v^i_{y_2} \end{bmatrix} = \begin{bmatrix} v^n \\ v^n_{y_L} \\ v^n_{y_2} \end{bmatrix} \quad S = S_{y_2} \cdot S_L$$

$S v^i$, $v^i_{y_L} = v^n_{y_2}$ $S_{y_2} \cdot v^n_{y_2} = v^i_2$

$v^i_{cir} = v^n_{L\,cir} = v^i_L$

$v^n_L = S_{2} \cdot v^i_L = v^i_{\text{port 2 of circulator}} = v^n_{\text{port 3 of circulator}} = v^i_{y_2}$

$S_{y_2} v^i_{y_2} = v^n_{y_2} = v^n \Rightarrow$ gives above product