EE610

10/15/15

Note Title 10/15/2015

$$R(s) = \frac{k Z(s) - s Z(k)}{k Z(k) - s Z(s)}$$ is PR if $Z(s)$ is & k is real & >0

for Bott-Duffin synthesis use this:

solve for $(k Z(k) - s Z(s)) R(s) = k Z(s) - s Z(k)$

$$Z(s)\left(-s R(s) - k\right) = -s Z(k) - k Z(k) R(s)$$

$$Z(s) = \frac{s \cdot Z(k) + k Z(k) R(s)}{k + s R(s)} = \frac{1}{\frac{k + s R(s)}{s Z(k)}} + \frac{1}{\frac{k + s R(s)}{k Z(k) R(s)}}$$

$$\Rightarrow \quad = \frac{1}{\frac{1}{s \cdot \frac{Z(k)}{k}} + \frac{1}{Z(k)} R(s)} + \frac{1}{\frac{1}{Z(k) R(s)} + \frac{s}{k Z(k)}}$$

$$Y = \frac{1}{Ls} + \frac{1}{\frac{k \cdot s}{s^2 + \omega_i^2} + R_2}$$

before using $R(s) = Z_{in}$ extract all poles and zeroes on $j\omega$ axis. on $j\omega$ axis, real part has a minimum of Z after remove poles, $Z(j\omega) = R(\omega) + jX(\omega) \Rightarrow R(\omega) = \text{minimum}$

subtract out $R(\omega_1) \geq 0 \Rightarrow$ positive (if >0) resistance
$\Rightarrow$ passive

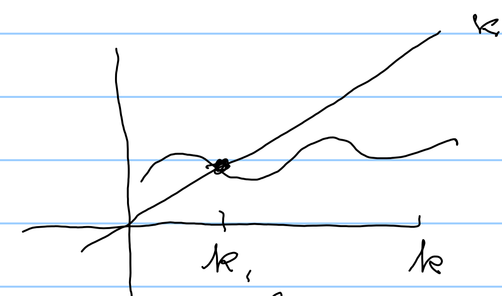
gives $z(s)$ that is minimum $\Rightarrow PR$, no poles or zeroes on $j\omega$ axis & $z(j\omega_1) = jX_1$

apply $R(s)$ to this minimum $z(s)$

$$R(j\omega_1) = \frac{k\, z(j\omega) - j\omega_1 z(k)}{k\, z(k) - j\omega_1 z(j\omega_1)} = \frac{jkX_1 - j\omega_1 z(k)}{k\, z(k) - j\omega_1 \cdot jX_1} = \frac{j(kX_1 - \omega_1 z(k))}{k\, z(k) + \omega_1 X_1}$$

assume $X_1 > 0$ (if not use denominator)

choose $kX_1 = \omega_1 z(k) \Rightarrow k = \frac{\omega_1}{X_1} \cdot z(k)$



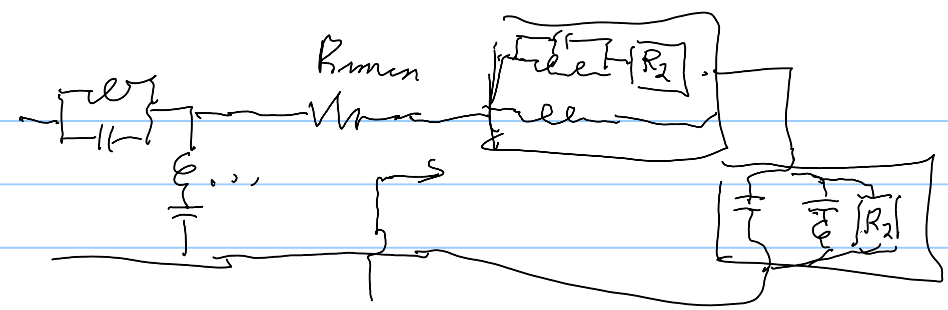
real

then $R(s)\Big|_{k=k_1}$ is $R(j\omega_1) = 0$

this means $y(s) = \frac{1}{R(s)}$ is PR with a pole at $s = \pm j\omega_1$

$y = \dfrac{ks}{s^2+\omega_1^2}$

$Z \longrightarrow$



apply $R(s)$ to this impedance

continue on R_2

R_2 is PR

synthesizes any PR function

as $\delta[R_2] < \delta[R] = \delta[Z]$

Bott-Duffin gives a transformerless synthesis of any PR function.

For a lossless PR $y(s) = 1/Z(s)$ cannot use $R(s)$ by k a zero of Even $y(s)$.

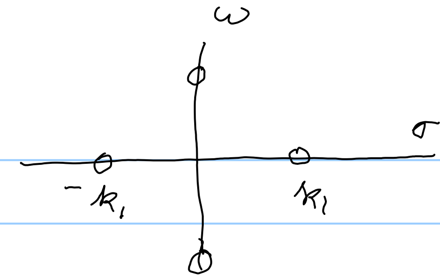
Get a cancellation of $k^2 - s^2$ if k is a zero of $Ev\, Z(s)$

$2\, Ev\, Z(s) = Z(s) + Z(-s)$ if k is a zero of $\mathcal{E}(Z(s))$ then $Z(k) = -Z(-k)$

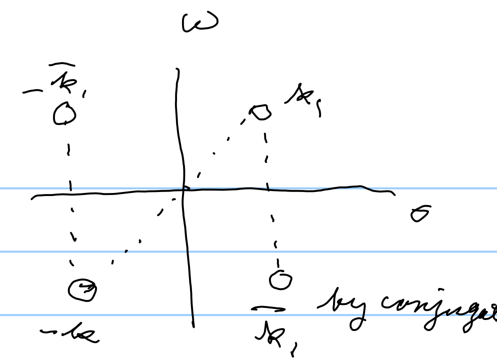
$$R(s) = \frac{kZ(s) - sZ(k)}{kZ(k) - sZ(s)}\Bigg|_{s=-k} = \frac{kZ(-k) - (-k)Z(k)}{kZ(k) - (-k)Z(-k)} = \frac{k(-Z(k)) + kZ(k)}{kZ(k) + (-Z(k))} = \frac{0}{0} \text{ if}$$

k is a zero of Even Z

zeros of $Ev\,z(s)$



also



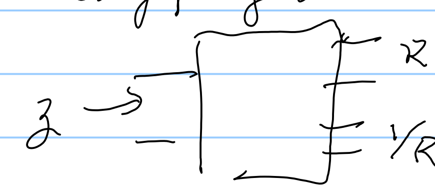
PR lossless $y(s) = -y(-s)$

$$= \sum \frac{2k_i s}{s^2 + \omega_i^2} + k_\infty s + \frac{k_0}{s} \Rightarrow \text{odd if lossless}$$

$\Rightarrow$ any $s = k$ will cancel $s^2 - k^2$ in a lossless PR $z \to R(s)$.

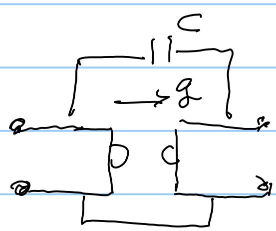
$\therefore$ need a coupling circuit to go $y(s)$ to $R(s)$

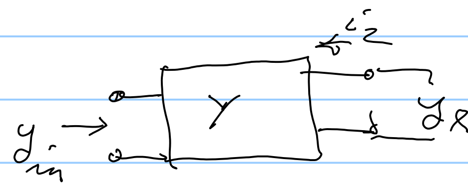
Bott-Duffin gives



$$R_z(\alpha) = \frac{k z(\alpha) - \alpha z(k)}{k z(k) - \alpha z(\alpha)} \Rightarrow \text{let } z = 1/y \Rightarrow \frac{k\frac{1}{y(\alpha)} - \alpha\frac{1}{y(k)}}{k\frac{1}{y(k)} - \alpha\frac{1}{y(\alpha)}} = \frac{k y(k) - \alpha y(\alpha)}{k y(\alpha) - \alpha y(k)}$$

also $1/R_z(\alpha)$ is PR $\Rightarrow$ $R_y(\alpha) = \dfrac{k y(\alpha) - \alpha y(k)}{k y(k) - \alpha y(\alpha)}$





$$y_{in} \approx \frac{g^2 + \alpha C y_\ell}{y_\ell + \alpha C}$$

$$Y = \begin{bmatrix} C\alpha & -C\alpha - g \\ -C\alpha + g & C\alpha \end{bmatrix}$$

$$-y_\ell v_2 = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$(-y_\ell - y_{22}) v_2 = y_{21} v_1$$

$$i_1 = y_{11} v_1 - \frac{y_{12} y_{21}}{y_\ell + y_{22}} v_1 = \frac{\Delta Y + y_\ell y_{11}}{y_\ell + y_{22}} = \frac{g^2 + \alpha C y_\ell}{y_\ell + \alpha C}$$

$$y_{in} \cdot y_\ell + \alpha C y_{in} = g^2 + \alpha C y_\ell$$

$$(y_{in} - \alpha C) y_\ell = g^2 - \alpha C y_{in} \Rightarrow y_\ell = \frac{g^2 - \alpha C y(\alpha)}{y(\alpha) - \alpha C} = C\left(\frac{g^2/C - \alpha y(\alpha)}{y(\alpha) - \alpha C}\right)$$

$$R_y = \frac{k\,y(k) - a\,y(a)}{k\,y(a) - a\,y(k)} \times y(k) \implies \text{gives } k,\ y(k) \text{ as element values}$$