Note Title 10/13/2015

EE 610

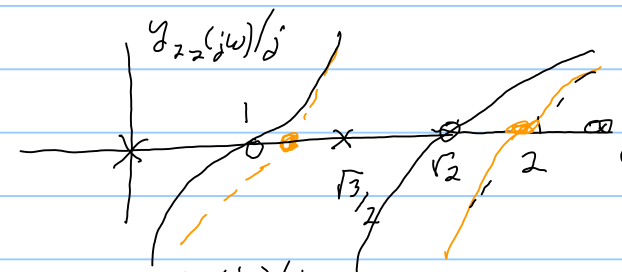
10/13/15

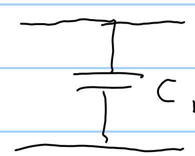
$$A_v(s) \overset{?}{=} \frac{?(-)(s^2+4)}{s^4+2s^3+3s^2+3s+2} = \frac{k\left((s^2+4)/(2s^3+3s)\right)}{1+\left(\frac{s^4+3s^2+2}{2s^3+3s}\right)} = \frac{-y_{21}}{1+y_{22}}$$

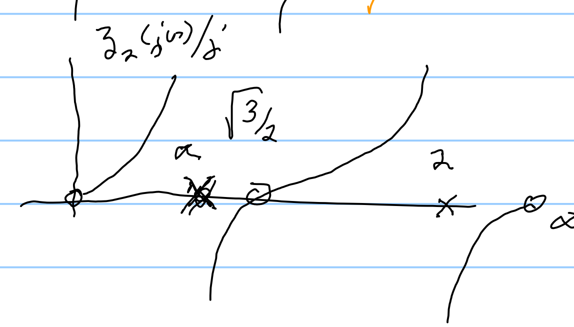
$R_L = 1\,\Omega$ normalized

$$y_{22} = \frac{(s^2+1)(s^2+2)}{2s(s^2+3/2)}$$

PR lossless

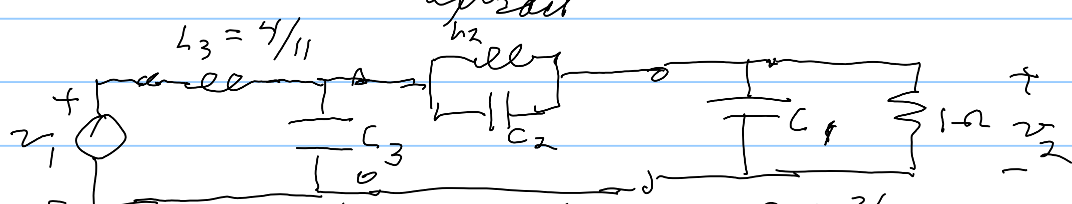






$$z_2 = \frac{1}{y_{22} - C_1 s} = \frac{2k_1 s}{s^2+4} + \frac{2k_2 s}{s^2+a^2}$$

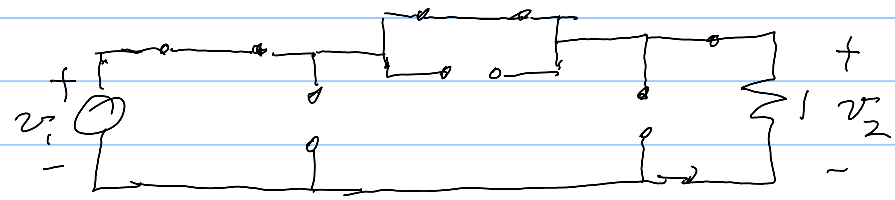
extract



$C_3 = 11/5$ $L_2 = 50/44$, $C_2 = \frac{11}{50}$ $C_1 = 3/10$

gives $-y_{21} = k\left(\frac{s^2+4}{2(s^2+3/2)s}\right.$

to find k: $A_v(0) = k \cdot 4/2$

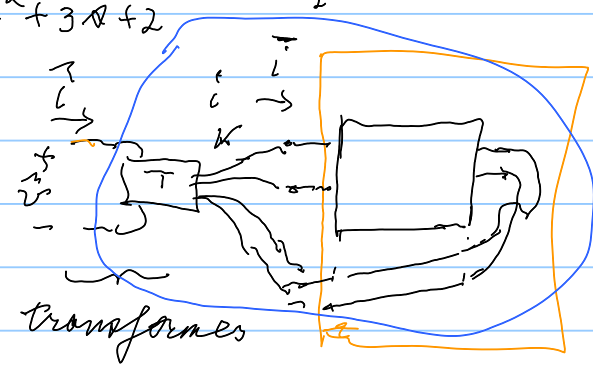


$\frac{v_2}{v_1}(0) = 1 = k \cdot 2 \Rightarrow k = 1/2$

$A_v = \frac{\frac{1}{2}(s^2+4)}{s^4+2s^3+3s^2+3s+2} = \frac{v_1}{v_2}$

$\begin{bmatrix} y_{11} & -(\ldots) \\ -(\ldots) & \end{bmatrix}$

Why is Y_{21} odd?



$i = Yv$

$v^T i + \hat{v}^T \hat{i} = 0$ = total power into transformer

$v = T\hat{v}$ $\quad i =$ $\quad v^T i = \hat{v}^T T^T i + \hat{v}^T \hat{i} = 0$ for all $\hat{v}$

$\hat{i} = -T^T i$ $\quad + \hat{v}^T \hat{i}$ $\quad \Rightarrow \hat{i} = -T^T i$

$$i = Yv \approx -i = +T\hat{i} = YT\hat{v} \Rightarrow \hat{i} = T^T Y T \hat{v}$$

$$\hat{Y} = T^T Y T$$

here $T = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$ as $\hat{v} = 1$ port voltage

$$\hat{Y} = \begin{bmatrix} t_1 & t_2 \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = t_1^2 y_{11} + t_2^2 y_{22} + t_1 t_2 (y_{12} + y_{21})$$

if Y is lossless 2-port then $\hat{Y}$ is a lossless positive-real function

if a ladder $\hat{Y} \Rightarrow PR$, $y_{21} = y_{12}$ if a ladder as the 2-port is reciprocal

$$Y_{in} = t_1^2 y_{11} + t_2^2 y_{22} + t_1 t_2 \times 2 y_{21} \Rightarrow$$ simple poles on $j\omega$ axis, positive & real residues

here y_{21} must have simple $j\omega$ axis poles with real residues

$\Rightarrow y_{21}$ is an odd function of s with its poles contained in y_{11} & y_{22} and near a pole, residue matrix is positive semi-definite $K = \begin{bmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{bmatrix}$

$k_{11} \geq 0$, $k_{22} \geq 0$, $\det = k_{11} k_{22} - k_{12}^2 \geq 0$

Richards' functions p. 361 $R(\alpha) = \dfrac{k\,z(\alpha) - \alpha\,z(k)}{k\,z(k) - \alpha\,z(\alpha)}$

$R(\alpha)$ is PR if $z(\alpha)$ is PR as $\alpha - k$ cancels $\delta[R(\alpha)] = \delta[z(\alpha)]$

unless another cancelation occurs

This happens if $z(k) = -z(-k)$ then $\alpha + k$ cancels

then $\delta[R(\alpha)] = \delta[z(\alpha)] - 1$