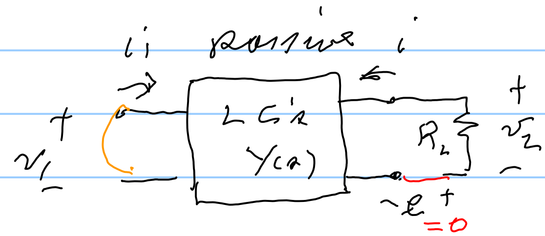
Note Title 10/8/2015

EE 610
10/08/15

Hurwitz polynomial: No zeros in $\sigma > 0$; $P(s)$, $s = \sigma + j\omega$
(& simple if on $j\omega$ axis) $\Rightarrow$ if none on $s = j\omega \Rightarrow$ strictly Hurwitz



$i_1 = y_{11} v_1 + y_{12} v_2$ if $v_1 = 0$

$i_2 = y_{21} v + y_{22} v_2$ $\quad i_2 = y_{22} v_2$

$\Uparrow$ driving point lossless $y(s)$

y_{22} is a lossless PR function

$e = R_L i_2 + v_2 \Rightarrow \dfrac{e - v_2}{R_L} = i_2 \Rightarrow e = \dfrac{i_2}{y_{22}} + R_L i_2$

$= \left(R_L + \dfrac{1}{y_{22}}\right) i_2 = \dfrac{1 + R_L y_{22}}{y_{22}} \cdot i_2$

$i_2 = \dfrac{y_{22}}{1 + R_L y_{22}} \cdot e$

$\Rightarrow$ if $e = 0 \Rightarrow i_2 \neq 0$ at zeros of $1 + R_L y_{22}$

can't be on $j\omega$ axis as R_L damps them.

set $1 + R_L y_{22} = \dfrac{n}{d} = 1 + R_L\, n_{22}/d_{22} = \dfrac{d_{22} + R_L n_{22}}{d_{22}} = \dfrac{P(s)}{d_{22}}$

$P(s)$ is Hurwitz $= Ev(P(s)) + Od(P(s)) = E(s) + O(s)$

$= E(s)\left[1 + O(s)/E(s)\right] = E(s)\left[1 + y_{22}(s)\right] = O(s)\left[1 + \dfrac{E}{O}(s)\right]$

$\Rightarrow$ if $P(s)$ is Hurwitz then can get a lossless PR $y(s)$ as $O(s)/E(s)$ or $E(s)/O(s)$

Ex: $P(s) = s^4 + 2s^3 + 3s^2 + 3s + 2 = (s^4 + 3s^2 + 2) + (2s^3 + 3s)$

$y(s) = \dfrac{2s(s^2 + 3/2)}{s^4 + 3s^2 + 2}$ is this lossless PR

1st Cauer: $z(s) = \dfrac{1}{y(s)}$

$2s^3 + 3s \,\big)\, s^4 + 3s^2 + 2 \quad \frac{1}{2}s$

$s^4 + \frac{3}{2}s^2$

$\frac{3}{2} = (3 - \frac{3}{2})s^2 + 2 \,\big)\, 2s^3 + 3s \quad \frac{4}{3}s$

$2s^3 + \frac{8}{3}s$

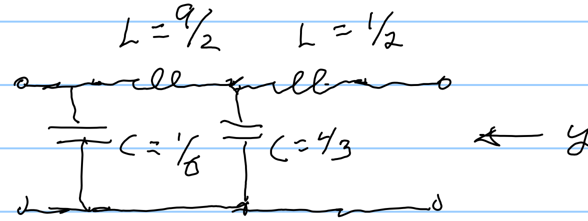
$\frac{9-8}{3}s = \frac{1}{3}s \,\big)\, \frac{3}{2}s^2 + 2 \quad \frac{9}{2}s$

$\frac{3}{2}s^2$

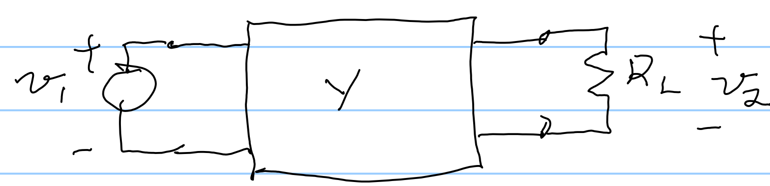
$2 \,\big)\, \frac{1}{3}s \quad \frac{1}{6}s$

$$y(s) = \cfrac{1}{\frac{1}{2}s + \cfrac{1}{\frac{4}{3}s + \cfrac{1}{\frac{9}{2}s + \cfrac{1}{\frac{1}{6}s}}}}$$

as this can result from a passive LC circuit, $P(s)$ is strictly Hurwitz



has all zeros for an y_{21} @ ∞ (as Cauer synthesis removes poles there)



$\frac{v_2}{v_1}$

$i_2 = y_{21} v_1 + y_{22} v_2 = -G_L v_2$

$\Rightarrow y_{21} v_1 = -(G_L + y_{22}) v_2$

$$\frac{v_2}{v_1} = \frac{-y_{21}}{G_L + y_{22}} = \frac{-y_{21} R_L}{1 + R_L y_{22}}$$

normalize $R_L = 1$

$$\frac{v_2}{v_1} = \frac{-y_{21}}{1 + y_{22}} = \frac{-m_{21}/d_{21}}{1 + m_{22}/d_{22}} = \frac{d_{22}}{d_{21}} \cdot \left(\frac{-m_{21}}{m_{22} + d_{22}} \right)$$

assume $d_{21} = d_{22}$ then $A_v(s) = \frac{v_2}{v_1} = \frac{-n_{21}}{n_{22} + d_{22}}$

Ex: $A_v(s) = \frac{-(s^2+4)}{s^4 + 2s^3 + 3s^2 + 3s + 2} = \frac{-(s^2+4)/(s^4+3s^2+2)}{1 + \frac{2s^3+3s}{s^4+3s^2+2}}$

$P(s) = (s^2+1)(s^2+2)$

$Y(s) = \frac{2s(s^2+3/2)}{(s^2+1)(s^2+2)}$ need a different synthesis than 1st Cauer

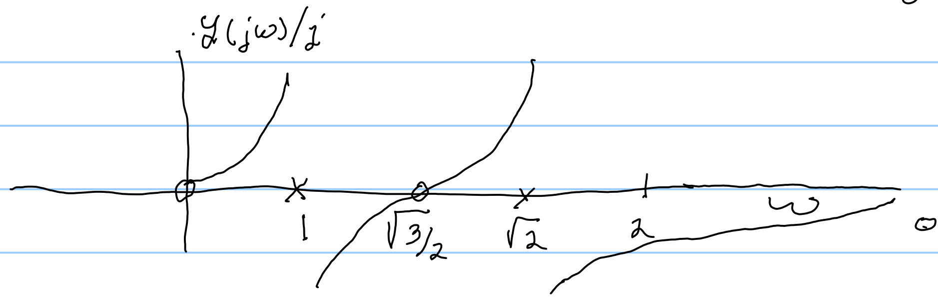
1st Cauer removes poles @ ∞

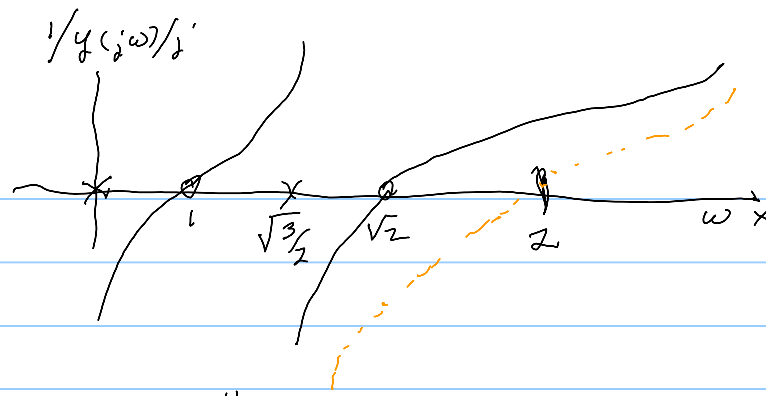
2nd Cauer " " " @ 0

1st Foster " " of $Z(s)$

2nd " " " of $Y(s)$

use partial pole removals to get zeros of Y_{21} on other points of $j\omega$ axis





desire to shift curve down by an extraction of part of the pole @ ∞

$$\frac{1}{y(\alpha)} - L\alpha = \frac{\alpha^4 + 3\alpha^2 + 2}{2\alpha^3 + 3\alpha} - L\alpha = \frac{P(\alpha)}{2\alpha^3 + 3\alpha} \quad , \quad P(\alpha^2 = -4) = 0$$

$$= \frac{\alpha^4 + 3\alpha^2 + 2 - 2\alpha^4 L - L3\alpha^2}{2\alpha^3 + 3\alpha} \Bigg|_{\alpha^2 = -4} = \frac{16 - 12 + 2 - 32L + 12L}{\alpha(-8+3)} \neq 0 = 0$$

$$= 6 - 20L = 0 \Rightarrow L = 3/10$$

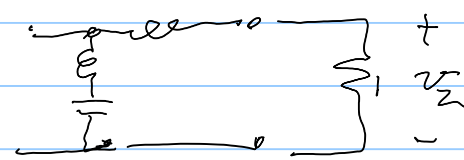
$$z_1(\alpha) = \frac{\alpha^4 + 3\alpha^2 + 2}{2\alpha^3 + 3\alpha} - \frac{3}{10}\alpha =$$

$$= \frac{\alpha^4 + 3\alpha^2 + 2 - 3/5\,\alpha^4 - \frac{9}{10}\alpha^2}{2\alpha^3 + 3\alpha}$$

$$= \frac{\frac{2}{5}\alpha^4 + \frac{21}{10}\alpha^2 + 2}{2\alpha^3 + 3\alpha} = \frac{(\alpha^2+4)(a\alpha^2 + b)}{(2\alpha^3 + 3\alpha)}$$

$$= \frac{(\alpha^2+4)(\frac{2}{5}\alpha^2 + 1/2)}{2\alpha^3 + 3\alpha}$$

$3/10 = L$



$$y_{shunt} = \frac{2k\alpha}{\alpha^2 + 4} + \ldots$$

$$
\begin{array}{r|l}
 & \frac{2}{5}\alpha^2 + \frac{1}{2} \\
\hline
\alpha^2+4 & \frac{2}{5}\alpha^4 + \frac{21}{10}\alpha^2 + 2 \\
 & \frac{2}{5}\alpha^4 + \frac{8}{5}\alpha^2 \\
\hline
 & \frac{5}{10}\alpha^2 + 2 \\
 & +2
\end{array}
$$

$$\Rightarrow \frac{1}{z_1} = \frac{2\alpha^3 + 3\alpha}{(\alpha^2+4)\frac{2}{5}(\alpha^2+5/4)} = \text{a new } y_2$$

$$y_2 = \frac{k_1\alpha}{\alpha^2+4} + \frac{k_2\alpha}{\alpha^2+5/4}$$

$$k_1 = y_2 \times \frac{(\alpha^2+4)}{\alpha}\Bigg|_{\alpha^2=-4} = \frac{2\alpha^2+3}{\frac{2}{5}(\alpha^2+5/4)}\Bigg|_{\alpha^2=-4} = \frac{-8+3}{\frac{2}{5}\left(\frac{-16+5}{4}\right)} = \frac{5}{2}\frac{(-5)}{(-15/4)} = \frac{25}{15/2} = \frac{50}{15}$$

created this poles