Note Title 10/6/2015

EE610

10/06/15

$$\frac{v_0}{v_i} = A_v(s) = \frac{N(s)}{D(s)} = \frac{P_1}{P_2} \cdot \frac{P_3}{P_4} \cdots \frac{P_{2m-1}}{P_{2m}}$$

all P_i of $\delta[P_i] \leq 2$

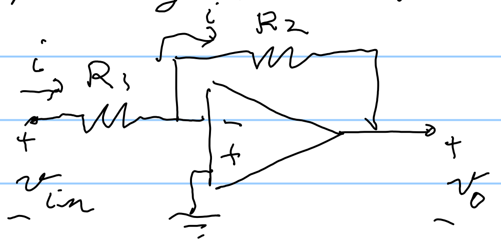
$0 \leq \delta[P_{2k}] - \delta[P_{2k-1}] \leq 1$

$$= A_{v_1} \cdots A_{v_{2m-1}} = A_{v_1} \cdots A_{v_i} \cdot A_{v_{2i+1}} \cdots A_{v_{2m-1}}$$

$\Uparrow$

need same conditions on A_i & $A_{v_{2i+1}}$

if couple by op-amps



$i = G_1 v_{in} = -G_2 v_0$

$$\frac{v_0}{v_{in}} = -\frac{G_1}{G_2}$$

need the current i small so we don't load the previous stage $\Rightarrow R_1$ large $\simeq 10\,\text{Meg}\Omega$.

Given $P(s) = s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0$

desire $P(s) = (s^2 + \alpha s + \beta)(s^{n-2} + b_{n-2} s^{n-2} + \dots + b_1 s + b_0)$

$= (s^2 + \alpha s + \beta) P_2(s)$

$$
\begin{array}{r|l}
 & s^{n-2} + \dots\ c_1 \\
\hline
s^2 + \alpha s + \beta & s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \\
 & \underline{s^n + \alpha s^{n-1} + \beta s^{n-2}} \\
 & (a_{n-1} - \alpha) s^{n-1} + (a_{n-2} - \beta) s^{n-2} + \dots a_1 s + a_0 \\
 & \overline{\qquad r_1 s + r_0 \qquad}
\end{array}
$$

$$\frac{P(s)}{s^2 + \alpha s + \beta} = s^{n-2} + \dots + c_1 + \frac{(\hat{r}_1 s + \hat{r}_0)}{s^2 + \alpha s + \beta} \cdot \; ; \; r_1 = \frac{\hat{r}_1 s}{s^2 + \alpha s + \beta}, \; r_0 = \frac{\hat{r}_0}{s^2 + \alpha s + \beta}$$

desire to force r_1 & r_0 to zero by choice of α & β

want $\alpha + \Delta\alpha$ & $\beta + \Delta\beta$ ~~to force~~ r_1 & r_0 ~~to zero~~

$$r_1 = r_{10} + \frac{\partial r_1}{\partial \alpha} \cdot \Delta\alpha + \frac{\partial r_1}{\partial \beta} \Delta\beta, \quad r_0 = r_{0_0} + \frac{\partial r_0}{\partial \alpha} \Delta\alpha + \frac{\partial r_0}{\partial \beta} \Delta\beta$$

desire r_1 & $r_0 = 0$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \doteq \begin{bmatrix} r_{10} \\ r_{00} \end{bmatrix} + \begin{bmatrix} \frac{\partial r_1}{\partial \alpha} & \frac{\partial r_1}{\partial \beta} \\ \frac{\partial r_0}{\partial \alpha} & \frac{\partial r_0}{\partial \beta} \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta\beta \end{bmatrix} + \cdots \Rightarrow \begin{bmatrix} \Delta\alpha \\ \Delta\beta \end{bmatrix} \doteq - \begin{bmatrix} \frac{\partial r_1}{\partial \alpha} & \frac{\partial r_1}{\partial \beta} \\ \frac{\partial r_0}{\partial \alpha} & \frac{\partial r_0}{\partial \beta} \end{bmatrix}^{-1} \begin{bmatrix} r_{10} \\ r_{00} \end{bmatrix}$$

Note these vary in s while α, β are constants
can choose s = any # for which the inverse exists
keep up. should converge.

~ – ~ –

Cauer synthesis from continued fraction expansion
for lossless PR functions

$$Y(s) = -Y(-s) \qquad Y(s) = \text{Even } Y(s) + \text{Odd } Y(s)$$

$$= \underbrace{\left[\frac{Y(s) + Y(-s)}{2}\right]}_{0 \text{ for lossless}} + \left[\frac{Y(s) - Y(-s)}{2}\right]$$

$\Rightarrow$ lossless PR is odd in s

$$Y(s) = \frac{s\, N(s^2)}{D(s^2)} \quad \text{or} \quad \frac{1}{s}\,\frac{N_1(s^2)}{D_1(s^2)}$$

Example $y(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+3)}$ & $y(s) = \frac{s(s^2+2)}{(s^2+1)} = \frac{s^3+2s}{s^2+1}$

1st Cauer removes poles at ∞

$s^2+1 \,\big)\, s^3+2s$ quotient s

s^3+s

$\frac{s}{s^2+1} \Rightarrow \frac{s^2+1}{s} \Rightarrow s \,\big)\, s^2+1$ quotient s

s^2

$1 \,\big)\, s$

$\Rightarrow s^2+1 \,\big)\, s^3+2s$ quotient $s \leftarrow y_1$

s^3+s

$s \,\big)\, s^2+1$ quotient $s \leftarrow z_2$

s^2

$1 \,\big)\, s$ quotient $s \leftarrow y_3$

s

$0 \leftarrow y_4$

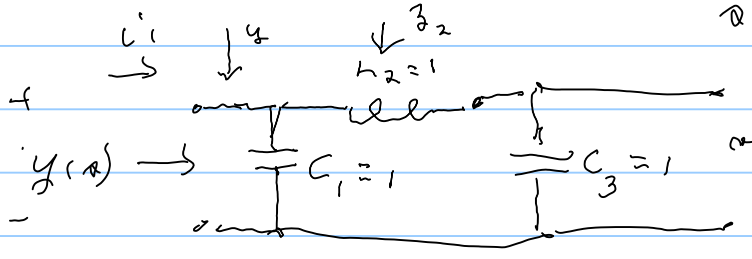
$$y(s) = \frac{s^3+2s}{s^2+1} = s + \cfrac{1}{s + \cfrac{1}{s+0}}$$

$\leftarrow y_1$; $z_1 = \frac{1}{y_1} = \frac{1}{s}$

$\leftarrow z_2$

$\leftarrow y_3$

& y_4

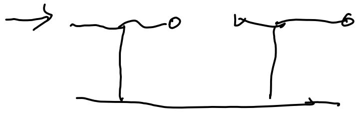


$+ \; v_2 \; -$ $\frac{v_2}{i_1} = \frac{k}{s^3+\cdots}$ by physical reasoning

$C's$ = shorts $\Big\}$ at $s=\infty$

$L's$ = opens

$z_L(s) = sL$, $z_C(s) = \frac{1}{sC}$ $\uparrow$ 0 @ $s=\infty$



v_2 is stopped 3 ways

$$y(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+3)} = \frac{s^3+2s}{s^4+4s^2+3}$$ no pole @ ∞

start with $z(s) = \frac{1}{y(s)}$ & divide highest powers of s into highest powers of s

$$z(s) = \frac{1}{y(s)} = \frac{s^4+4s^2+3}{s^3+2s},$$

$s \leftarrow z_1$

$s^3+2s \,\big)\, s^4+4s^2+3$

s^4+2s^2

$2s^2+3 \,\big)\, s^3+2s \quad \frac{1}{2}s \leftarrow y_2$

$s^3+\frac{3}{2}s$

$(2-3/2)s = \frac{1}{2}s \,\big)\, 2s^2+3 \quad 4s \leftarrow z_3$

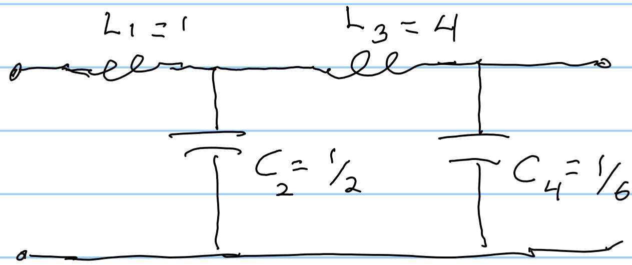
$2s^2$

$3 \,\big)\, \frac{1}{2}s \quad \frac{1}{6}s \leftarrow y$

$$y(s) = \frac{s^3+2s}{s^4+4s^2+3} = \cfrac{1}{s + \cfrac{1}{\frac{1}{2}s + \cfrac{1}{4s + \cfrac{1}{\frac{1}{6}s}}}}$$

1st Cauer

uses minimum # of reactive elements (L's & C's)



2nd Cauer → removes poles at $s = 0$, ÷ lowest power of s into lowest power of s

$$z(s) = \frac{1}{y(s)} = \frac{3+4s^2+s^4}{2s+s^3}$$

$$2s+s^3 \overline{\big)\, 3+4s^2+s^4}\quad \frac{3}{2s}$$

$$\underline{3+\frac{3}{2}s^2}$$

$$\frac{5}{2}s^2+s^4 \overline{\big)\, 2s+s^3}\quad \frac{4}{5s}$$

$$\underline{2s+\frac{4}{5}s^3}$$

$$\frac{1}{5}s^3 \overline{\big)\, \frac{5}{2}s^2+s^4}\quad \frac{25}{2s}$$

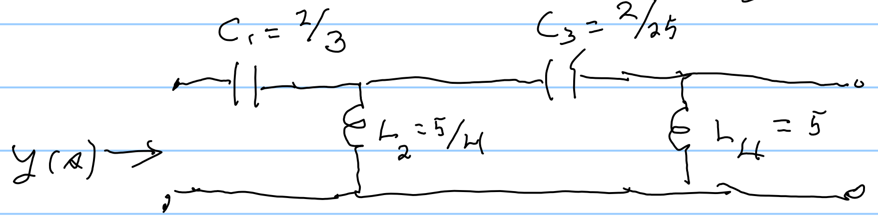
$$\underline{\frac{5}{2}s^2+}\; s^4 \overline{\big)\, \frac{1}{5}s^3}\quad \frac{1}{5s}$$

$$\underline{\frac{1}{5}s^3}$$

$$y(s) = \cfrac{1}{\cfrac{3}{2s} + \cfrac{1}{\cfrac{4}{5s} + \cfrac{1}{\cfrac{25}{2s} + \cfrac{1}{\cfrac{1}{5s} + 0}}}}$$

continued fraction expansion
about $s = 0$
⇒ 2nd Cauer

" y_4 = open



$$\frac{v_o}{v_i} = \frac{Ks^4}{s^4 + \ldots}$$