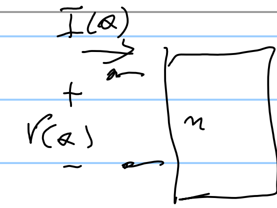
EE610

10/01/15

Note Title 10/1/2015

Lossless synthesis of passive 1-ports



$I = YV$

$$P_{ave}(j\omega) = \mathrm{Re}\, V^{*T}(j\omega) I(j\omega) = \left[V^{*T} I + (V^T I^*)^T \right]/2$$
$$= \left[V^{*T} I + I^{*T} V \right]/2$$

$$\left[V^{*T} Y(j\omega) V + V^{*T} Y^{*T} V \right]/2 = 0 \text{ if lossless}$$

$$V^{*T}\left[\frac{Y(j\omega) + Y^{*T}(j\omega)}{2} \right] V = 0 \Rightarrow Y(j\omega) + Y^{*T}(j\omega) \text{ is } = 0_n$$

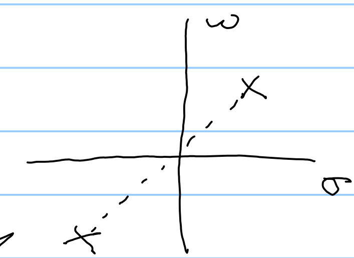
If rational & PR means real for real $s > 0$

$$(1\times 1):\quad y(s) = \frac{n(s)}{d(s)} = \frac{n_n s^n + n_{n-1} s^{n-1} + \dots + n_0 s^0}{s^m + d_{m-1} s^{m-1} + \dots + d_0 s^0} \qquad n_i \,\&\, d_i \text{ real}$$

if $y(j\omega)$ is conjugated $\Rightarrow y^*(j\omega) = y(-j\omega)$ as

$$y(j\omega) + y^*(j\omega) = \left[y(s) + y(-s) \right]_{\text{if } s = j\omega} = 0 \text{ for } s = j\omega$$

$\Rightarrow 0$ for all s by analytic continuation & $= 0$ on a dense set of $s = j\omega$ $\qquad y(s) = -y(-s)$



$s = \sigma + j\omega$

if a pole of $y(-s)$ it is also a pole of $y(s)$ $\Rightarrow$ no poles of a lossless $y(s)$ in the LHP or RHP also true for $z(s) = 1/y(s)$

$\Rightarrow$ also no zeros of $y(s)$ ($\&\ z(s)$) in LHP or RHP

These are simple as a multiple pole leads to instabilities

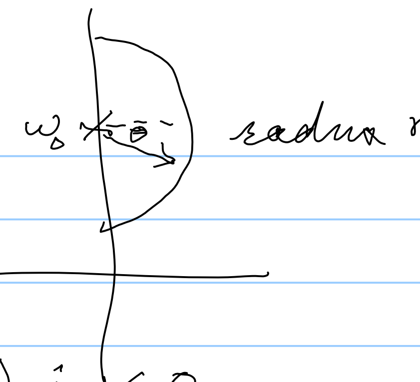
$$y(s) \approx \frac{k}{(s - j\omega_0)^m} + \dots$$

$\mathrm{Re}\, y(s) \geq 0$ in the RHP

$$\mathrm{Re}\, y(s) \Big| = \frac{|k|}{|s - j\omega_0|^m}\ \frac{1}{\mathrm{Re}}$$

near $s = j\omega$ but with $\sigma > 0$

$$y(s) = \frac{|k| e^{j\measuredangle k}}{\left(|r| e^{j\measuredangle\theta}\right)^m} = \frac{|k|}{r} e^{j(\measuredangle k - m\measuredangle\theta)}$$



$$-\frac{\pi}{2} < \angle\theta < \pi/2$$

Re $y(s)$ will go negative if $\cos(\measuredangle k - m\measuredangle\theta)$ is < 0

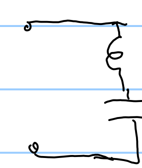
means $m = 1$ & $\measuredangle k = 0$

near a pole $\quad y(s) = \dfrac{k}{s - j\omega_0} + \dfrac{k}{(s + j\omega_0)} + \cdots$ — same for all poles

$$y(s) = \frac{2ks + j\omega_0 k - kj\omega_0}{(s + j\omega)(s - j\omega)} + \cdots = \frac{2ks}{s^2 + \omega_0^2} + \cdots \quad \frac{k_0}{s} + k_\infty s$$

Lossless $\quad y(s) = \dfrac{k_0}{s} + k_\infty s + \displaystyle\sum_{i=1}^{m-1} \frac{2k_i s}{s^2 + \omega_i^2}$

$$y_1 = \frac{2k_1 s}{s^2 + \omega_1^2} = \frac{1}{\dfrac{s^2}{2k_1 s} + \dfrac{\omega_1^2}{2k_1 s}} = \frac{1}{\dfrac{s}{2k_1} + \dfrac{1}{\frac{2k_1}{\omega_1^2} s}} = \frac{1}{z_{10} + \dfrac{1}{y_{20}}}$$

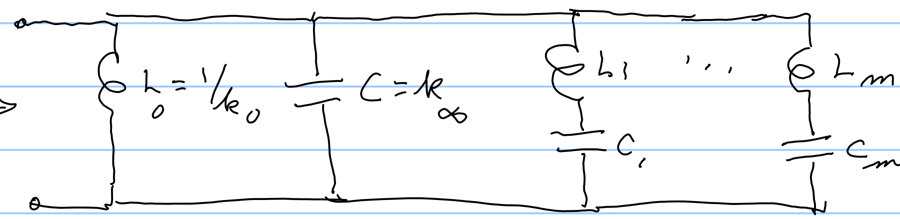


$L = 1/2k_1$

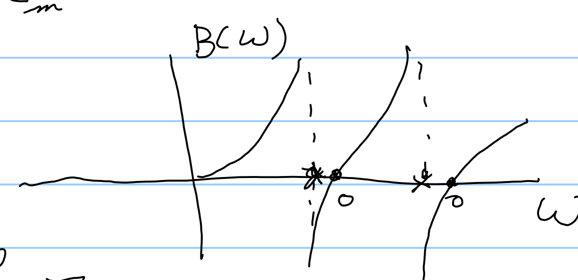
$C = \dfrac{2k_1}{\omega_1^2}$

synthesizes $y(s)$ by parallel series resonant circuits

$y(s) \rightarrow$



2nd Foster realization



$$y(j\omega) = j\,B(\omega)$$

$\dfrac{d\,B(\omega)}{d\omega}$; $\quad B(\omega) = -\dfrac{k_0}{\omega} + k_\infty \omega + \sum_{i=1} \dfrac{2k_i\omega}{-\omega^2+\omega_i^2}$

$$\frac{d\,B(\omega)}{d\omega} = -\left(\frac{-1}{\omega^2}\right)k_0 + k_\infty + \sum \frac{2k_i}{(-\omega^2+\omega_i^2)} + 2\omega \times \frac{k_i}{(-\omega^2+\omega_i^2)^2}(-2\omega)$$

$$= \frac{1}{\omega^2}k_0 + k_\infty + \sum \frac{2k_i\left[(-\omega^2+\omega_i^2) + 2\omega^2\right]}{(-\omega^2+\omega_i^2)^2}$$

$$= \frac{1}{\omega^2}k_0 + k_\infty + \sum \frac{2k_i(\omega^2+\omega_i^2)}{(-\omega^2+\omega_i^2)^2} > 0$$

$\Rightarrow$ poles & zeros alternate ; $y(s) = -y(-s) \Rightarrow$ odd

$$y(s) = \frac{s(s^2+2)}{(s^2+1)} \quad \text{is PR lossless}$$

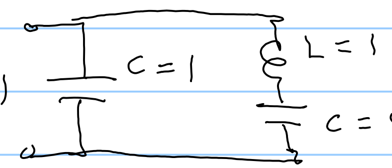
$$= s + \frac{2k_1 s}{s^2+1} \qquad \times \frac{s^2+1}{s} \Rightarrow s^2+2 = y(s) \times \frac{s^2+1}{s} = \frac{s(s^2+1)}{s} + 2k_1$$

set $s^2 = -1 \Rightarrow 2k_1$

$$s^2+2\Big|_{s^2=-1} = -1+2 = 2k_1 \quad k = 1/2 > 0$$

$$= s + \frac{s}{s^2+1}$$

$$= \frac{s^3+s+s}{s^2+1} = \frac{s(s^2+2)}{s^2+1} \Rightarrow y(s)$$



$$y_1 = \frac{s}{s^2+1} = \frac{1}{s+1/s}$$

1st Foster $\Rightarrow$ dual $\quad z(s) = \frac{\hat{k}_0}{s} + \hat{k}_\infty s + \sum_{i=1}^{\hat{n}} \frac{2\hat{k}_i s}{s^2+\hat{\omega}_i^2}$

$$z(s) = \frac{s^2+1}{s(s^2+2)} = \frac{\hat{k}_0}{s} + \frac{2\hat{k}s}{s^2+2} = \frac{\hat{k}_0}{s} + \frac{\hat{k}}{s+j\sqrt{2}} + \frac{\hat{k}}{s-j\sqrt{2}}$$

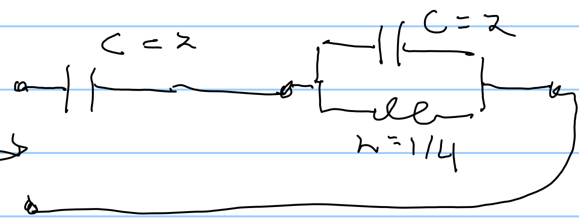
$$k_0 \Rightarrow \quad s z(s) = \frac{s^2+1}{s^2+2} = \hat{k}_0 + \frac{2\hat{k}s^2}{s^2+2} \Rightarrow \hat{k}_0 = s z(s)\Big|_{s=0} = \frac{s^2+1}{s^2+2}\Big|_{s=0} = 1/2$$

$\hat{k} \Rightarrow \frac{s^2+2}{2s}\cdot Z(s) = \frac{s^2+1}{2s^2} = \frac{k_0(s^2+2)}{2s^2} + \hat{k}$ set $s^2 = -2$

$\Rightarrow \frac{-2+1}{2(-2)} = \frac{-1}{-4} = \frac{1}{4} = 0 + \hat{k} \Rightarrow \hat{k} = 1/4$

$Z(s) = \frac{s^2+1}{s(s^2+2)} = \frac{1}{2s} + \frac{2\times 1/4\, s}{s^2+2} = \frac{1}{2s} + \frac{1/2\, s}{s^2+2} = \frac{1}{2s} + \frac{1}{2s + 4/s}$

$Z(s) \rightarrow$



1st Foster form

$\delta[y(s)]$ = degree $= 3 = \delta[Z(s)] \Rightarrow$ need at least 3 components, so we used the minimum number = canonical