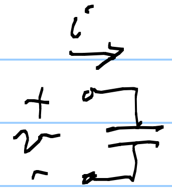
Note Title 9/29/2015

EE610
09/29/15

$\dot{x} = Ax + Bu$ $\qquad T(s) = D + C(s 1_k - A)^{-1} B$, $T(\infty) = D$

$y = Cx + Du$ $\qquad$ does not allow a pole at ∞

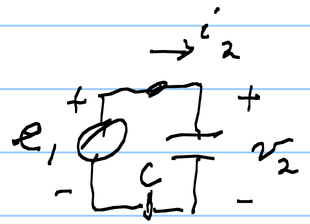
$+ sE + s^2 F \ldots$

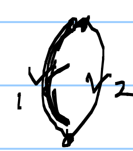
But 

$i(s) = sC \cdot v(s) \Rightarrow T(s) = sC$ has a pole at ∞

allowed in $E\dot{x} = Ax + Bu$, E possibly singular

$y = Cx$





KCL $0 = [1 \quad 1] \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}_b \Rightarrow v_b = E^T v_t = \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_t$

KVL $0 = [-1 \quad 1] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_b \Rightarrow i_b = \sigma^T i_\ell = \begin{bmatrix} -1 \\ 1 \end{bmatrix} i_\ell$

$v_b = e + v = \begin{bmatrix} e_1 \\ 0 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ v_2 \end{bmatrix}$

$i_b = j + i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}_b = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

$b = 2$, A, B for $Av = Bi$ are $b \times b = 2 \times 2$

$$\begin{bmatrix} 1 & 0 \\ 0 & \alpha C \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$-\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} e_1 \\ 0 \end{bmatrix} - \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= \begin{bmatrix} e_1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_t^{\,b}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = i_b = \begin{bmatrix} -1 \\ 1 \end{bmatrix} i_\ell$$

$$-\begin{bmatrix} 1 & 0 \\ 0 & \alpha C \end{bmatrix} \left\{ \begin{bmatrix} e_1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_t \right\} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} i_\ell$$

$$-\begin{bmatrix} -1 & 0 \\ 0 & -\alpha C \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_t - \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} i_\ell = \begin{bmatrix} -1 & 0 \\ 0 & -\alpha C \end{bmatrix} \begin{bmatrix} e_1 \\ 0 \end{bmatrix} = \begin{bmatrix} +e_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} +1 \\ +\alpha C \end{bmatrix} v_t - \begin{bmatrix} 0 \\ 1 \end{bmatrix} i_\ell = \begin{bmatrix} +1 & 0 \\ +\alpha C & -1 \end{bmatrix} \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = \begin{bmatrix} +e_1 \\ 0 \end{bmatrix} \qquad x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ C & 0 \end{bmatrix} \alpha x = \begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} e_1$$

$i = y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$ has $i = \alpha C v$, $e_c = v$

1st eq. $x_1 = e_1$, $\quad$ c.a. $x_1 = + x_2$; $\quad$ c.a. $e_1 = + i_2 = + i_2$

for a double pole @ ∞ can add another branch in parallel with $i_3 = \alpha i_2$, etc.,

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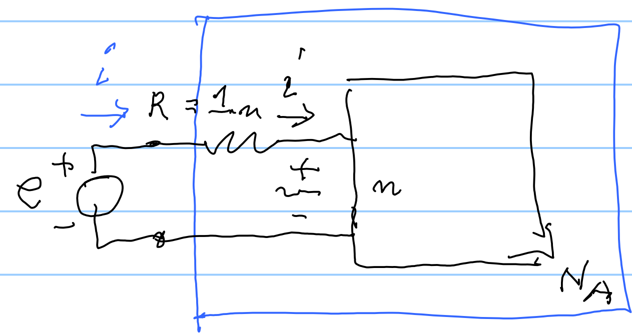
scattering



$i = Y_A e$

$e = v + i \Rightarrow v = e - i$

$v = 1_n \cdot e - Y_A \cdot e = (1_n - Y_A) e$

$$A v = B i$$

$$A (1_n - Y_A) e = B Y_A e$$

also $2 v^i = v + i$

$2 v^r = v - i$

$$\Rightarrow v = v^i + v^r, \quad i = v^i - v^r$$

$$A (v^i + v^r) = B (v^i - v^r) \Rightarrow (A + B) v^r = (B - A) v^i$$

$$v^r = (B + A)^{-1} (B - A) v^i$$

$$S = (B+A)^{-1}(B-A) \quad \text{for } v^r = S v^i$$

choose $A = Y_A$ & $B = (1_n - Y_A)$ then $A(1_n - Y_A) = Y_A - Y_A^2$
$= B(Y_A) =$

$$Y_A v = (1_n - Y_A) i \Rightarrow A = Y_A,\ B = (1_n - Y_A)$$

$\Rightarrow$ S goes with this A & B

$$S = (B+A)^{-1}(B-A) = (1_n - Y_A + Y_A)^{-1}(1_n - Y_A - Y_A) = 1_n - 2Y_A$$

$S = 1_n - 2Y_A$ one way to find S

if Z exists then $v = Z i \Rightarrow A = 1_n,\ B = Z$

$$S = (1_n + Z)^{-1}(Z - 1_n)\ ;$$

Ex: inductor $Z = sL$, $v = sL i$

$$S = (1 + sL)^{-1}(sL - 1) = \frac{(sL-1)}{(sL+1)}$$

$S(\infty) = 1$ no pole

if Y exists $\Rightarrow i = Y v$

$$A = Y,\ B = 1_n \Rightarrow S = (Y + 1_n)^{-1}(1_n - Y)$$

$$S = (Y + 1_n)^{-1}(1_n - Y) = (Z + 1_n)^{-1}(Z - 1_n) = 1_n - 2Y_A$$

In time domain: N is passive if $\int_{-\infty}^{t} p(\tau)\,d\tau = \int_{-\infty}^{t} v^T(\tau)\, i(\tau)\,d\tau \geq 0$

$$\int_0^t e^{T}(\tau)\, e(\tau)\,d\tau \geq 0 \text{ \& finite if } e_i \text{ are in } L_2 = \text{square integrable}$$

$$= \int_0^t [v + i]^T [v + i]\,d\tau = \int_0^t \Big[\underbrace{v^T(\tau) v(\tau) + i^T(\tau) i(\tau)}_{\geq 0} + \underbrace{2 v^T(\tau) i(\tau)}_{\geq \text{positive if passive}}\Big]\,d\tau$$

& finite [in L_2 by assumption]

$\Rightarrow$ if e is in L_{2_n} then v & i are also if N is passive

Look at $P_{in}(j\omega) = \operatorname{Re} v^{T*}(j\omega)\, i(j\omega)$

$$= \operatorname{Re}\Big[(v^i(j\omega) + v^r(j\omega))^{T*}(v^i(j\omega) - v^r(j\omega))\Big]$$

$$= \operatorname{Re}\Big[v^{iT*}(j\omega) v^i(j\omega) - v^{rT*}(j\omega) v^r(j\omega) + \underbrace{v^{rT*}(j\omega) v^i(j\omega) - v^{iT*}(j\omega) v^r(j\omega)}_{\text{cancel as have same Re part}}\Big]$$

$$= v^{iT*}(j\omega) v^i(j\omega) - v^{rT*}(j\omega) v^r(j\omega)$$

$$P_{in}(j\omega) = \mathrm{Re}\, v^{T*}(j\omega)\, i(j\omega) = v^{iT*}(j\omega)\, v^{i}(j\omega) - v^{rT*}(j\omega)\, v^{r}(j\omega)$$

$$= \mathrm{Re}\, v^{T*}(j\omega)\, Y(j\omega)\, v(j\omega) = v^{iT*}\left[1_n - S^{T*}(j\omega)\, S(j\omega)\right] v^{i}(j\omega)$$

$$= \frac{1}{2}\left[v^{T*}(j\omega)\left[Y(j\omega) + Y^{T*}(j\omega)\right] v(j\omega)\right]$$

Conditions for passivity

1) $Y(s)$ (and $Z(s)$) is <u>positive-real</u> $\Longleftrightarrow$ 2) $S(s)$ is <u>bounded real</u>

iff

a) $Y(s)$ is real for s real & positive
(coefficients real if $Y(s)$ is rational = ratio of 2 polynomials)

(note $\sqrt{s}$ is positive-real)

b) $Y(s)$ has no singularity in $\mathrm{Re}\, s > 0$ [the circuit is stable]

c) $\mathrm{Re}(Y(s))$ is positive semi-definite in $\sigma > 0$

a) also $S(s)$ is real for real positive s

b) $S(s)$ has no singularity in $\mathrm{Re}\, s = \sigma > 0$

c) $1_n - S^{*T}(s) S(s)$ is positive semi-definite in $\sigma > 0$

If $P(j\omega) = 0 \Rightarrow Y^{T*}(j\omega) + Y(j\omega) = 0$ (for almost all ω)

if PR $\quad Y^{T*}(j\omega) = Y^{T}(-j\omega)$

$$\Rightarrow Y^{T*}(j\omega) + Y(j\omega) = Y^{T}(-j\omega) + Y(j\omega)$$

$$= \left(Y^{T}(-a) + Y(a)\right)\Big|_{a = j\omega \Rightarrow \omega = a/j}$$