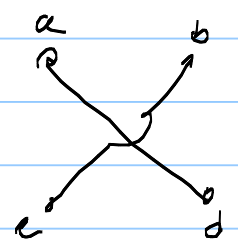
Note Title

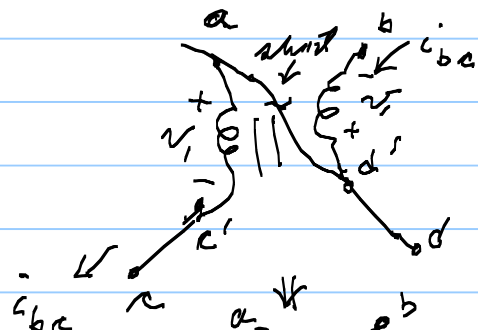
EE610

09/24/15

9/24/2015

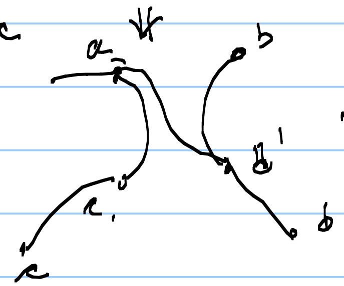
How to get a planar graph:





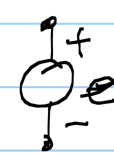
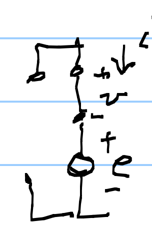
$v_{c'} = v_b$ as $v_1 + (-v_1) = 0$

current in a goes to d'



no crossing lead

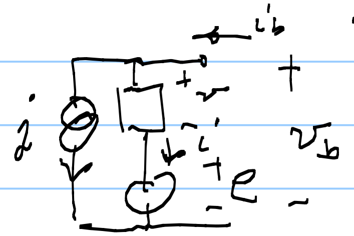
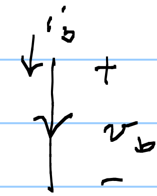
$\Rightarrow$ allows a graph with no crossing leads

What to do if a branch is  $\Rightarrow$ 

$v = 0$

$Av = Bi$

$[1][v] = [0][i]$

 = 

$v_b = v + e$

$i_b = i + j$

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_b \end{bmatrix} \qquad v_b = \begin{bmatrix} v_{1b} \\ v_{2b} \\ \vdots \\ v_{bb} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_b \end{bmatrix}_b \qquad \begin{aligned} v_b &= \mathcal{C}^T v_t \\ i_b &= \mathcal{J}^T i_\ell \end{aligned}$$

if linear then $Av = Bi$

$$v = v_b - e \; ; \quad Av = Av_b - Ae = Bi = Bi_b - Bj$$

$$i = i_b - j \qquad Av_b - Bi_b = Ae - Bj$$

$$A\mathcal{C}^T \cdot v_t - B\mathcal{J}^T i_\ell = Ae - Bj$$

$$\left[Ae^T \,\vdots\, -B\mathcal{J}^T \right] \underbrace{\begin{bmatrix} v_t \\ i_\ell \end{bmatrix}}_{\text{a } b\text{-vector}} = Ae - Bj \Rightarrow \text{a } b\text{-vector}$$

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scattering matrices

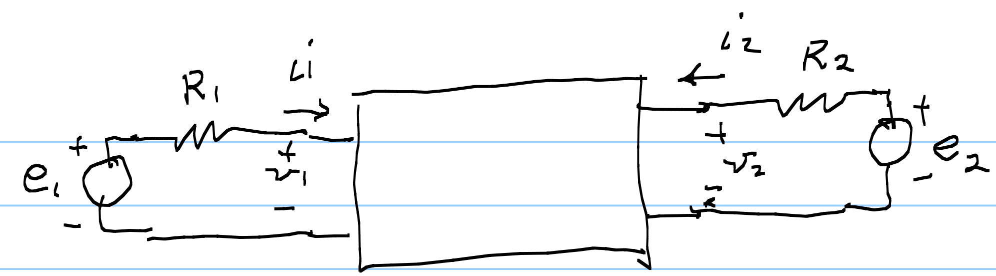
$v^i \Rightarrow$ incident voltages, v^r = reflected voltages

$2v^i = v + Ri \qquad 2v^r = v - Ri \qquad R \sim Z_0$ of a transmission line

$2v = 2v^i + 2v^r \Rightarrow v = v^i + v^r$

$2Ri = 2v^i - 2v^r \Rightarrow Ri = v^i - v^r$

$i = G(v^i - v^r)$



$$\Rightarrow \; e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$e_1 = v_1 + R i_1 \qquad e_2 = v_2 + R_2 i_2$$

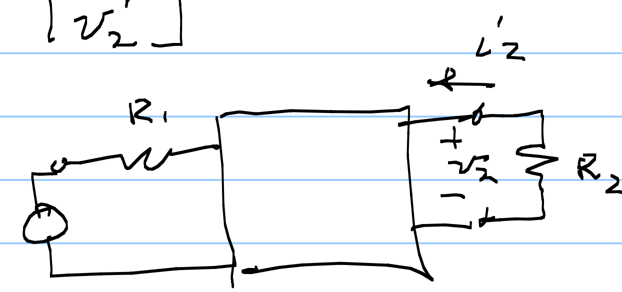
$$e = v + \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} i$$

$$= v + R i = 2v^i \Rightarrow v^i = e/2$$

$$v^r = S v^i = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_1^i \\ v_2^i \end{bmatrix} = \begin{bmatrix} v_1^r \\ v_2^r \end{bmatrix}$$

if $v_2^i = 0 \Rightarrow e_2 = 0 \Rightarrow$ a short

$$i_2 = -G_2 v_2$$



$$v_2^r = a_{21} v_1^i = \frac{v_2 - R_2 i_2}{2}$$

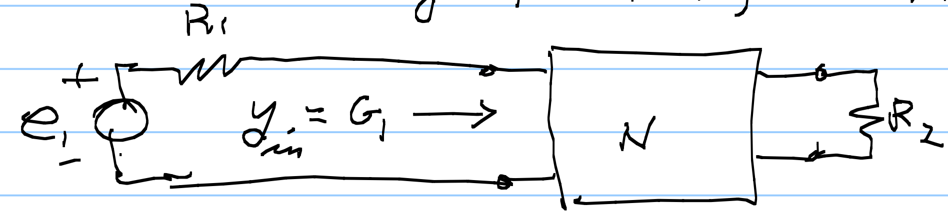
$$= \frac{v_2 + R_2 \cdot G_2 v_2}{2} = \frac{2v_2}{2} = v_2 = a_{21} v_1^i = a_{21} \frac{e_1}{2}$$

$$a_{21} = \frac{v_2}{v_1^i} = \frac{v_2}{e_1/2} = 2\frac{v_2}{e_1} \Rightarrow A_v = \frac{a_{21}}{2} = \text{voltage gain}$$

$$s_{11} = \left.\frac{v_1^r}{v_1^i}\right|_{v_2^i = 0} \qquad v_2^i = 0 = v_2 + R_2 i_2 \Rightarrow v_2 = -R_2 i_2 \text{ above case} = e_2 = 0$$

$$= \frac{(v_1 - R_1 i_1)/2}{(v_1 + R_1 i_1)/2} = \text{reflection coefficient}$$

$= 0$ if $i_1 = G_1 v_1$, $G_1 = 1/R_1$, call matched



$$v_1 \cdot i_1 + v_2 \cdot i_2 = \underset{(\text{into } N)}{P_{in}(t)} = v^T(t) \cdot i(t)$$

$$= (v^i + v^r)^T G (v^i - v^r)$$

$$= v^{iT} G v^i - \underbrace{v^{iT} G v^r + v^{rT} G v^i}_{} - v^{rT} G v^r$$

$$\underbrace{(v^{iT} G v^r)}^T = v^{rT} G^T v^i, \quad G^T = G \text{ as diagonal}$$

$$= v^{iT} G v^i - v^{rT} G v^r$$

if passive then energy in should be ≥ 0

if normalize $G = 1_2 \Rightarrow P_{in}(t) = v^{i^T} v^i - v^{r^T} v^r$

$$\mathcal{E}_N(t) = \int_{-\infty}^{t} P_{in}(\tau)\, d\tau = \int_{-\infty}^{t} \left[v^{i^T}(\tau) v^i(\tau) - v^{r^T}(\tau) v^r(\tau) \right] d\tau$$

$\Downarrow$ Parsevals

if $\mathcal{E}_N(t) \geq 0 \Rightarrow v^i \in L_2 \Rightarrow v^r \in L_2$

$$V^r(s) = S(s) V^i(s) \qquad \Rightarrow \int_{-\infty}^{\infty} \left[V^{i^{T*}}(j\omega) V^i(j\omega) - V^{r^{T*}}(j\omega) \cdot V^r(j\omega) \right] d\omega \geq 0$$

if passive

Parseval's theorem

$$\int \left[V^{i^{T*}} V^i - V^{i^{T*}} S^{*T}(j\omega) \cdot S(j\omega) V^i \right] d\omega \geq 0 \text{ if passive}$$

$$= \int V^{i^{T*}} \left[1_2 - S^{T*}(j\omega) S(j\omega) \right] V^i \, d\omega \geq 0 \Rightarrow 1_2 - S^{T*}(j\omega) S(j\omega) \geq 0^{2\times 2}$$

is positive semi-definite

$S(s)$ is real for real s

(if rational all coefficients are real

$S(s)$ is analytic in $\sigma \geq 0$, $s = \sigma + j\omega$)

$$1_2 - S^{T*}(s)S(s) \succeq 0 \quad \text{in } \sigma \geq 0$$

positive semidefinite

These 3 conditions define $S(s)$ as a bounded real matrix