Note Title 9/17/2015

EE 610
09/17/15

Equivalent circuits

$$E\dot{x} = Ax + Bu \qquad x = \text{semi-state} = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = b\text{-vector}$$
$$y = Cx$$

A is $b \times b$

1st premultiply by a P that is nonsingular

$$PE\dot{x} = PAx + PBu, \quad y = Cx$$

2nd $x = Q\hat{x}$, Q nonsingular

$$PEQ\dot{\hat{x}} = PAQ\hat{x} + PBu, \quad y = CQ\hat{x}$$

$$y = T(s)u, \quad T(s) = C(sE - A)^{-1}B = CQ(sPEQ - PAQ)^{-1}PB$$
$$= CQ([sPE - PA]Q)^{-1}PB$$
$$= CQQ^{-1}(sPE - PA)^{-1}PB$$
$$= C(P[sE - A])^{-1}PB$$
$$= C(sE - A)^{-1}P^{-1}PB$$
$$= C(sE - A)^{-1}B$$

if use P & Q to diagonalize E can get

1) $$\begin{bmatrix} I_c & 0 \\ 0 & 0_{b-c} \end{bmatrix} \dot{\hat{x}} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \hat{x} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \qquad \hat{x} = \begin{bmatrix} x_c \\ x_{b-c} \end{bmatrix}$$

2) $$y = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \hat{x}$$

3) $$I_c \dot{x}_c = A_{11} x_c + A_{12} x_{b-c} + B_1 u$$

4) $$-A_{22} x_{b-c} = A_{21} x_c + B_2 u$$

if A_{22} is nonsingular then we can state variable equation

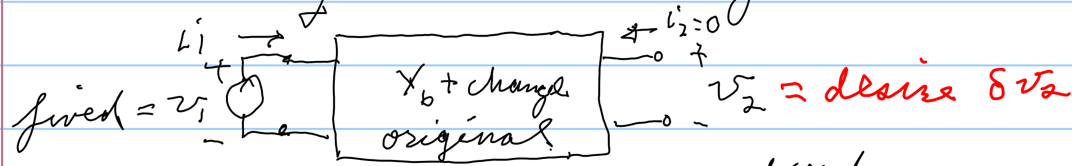
5) $$x_{b-c} = -A_{22}^{-1} A_{21} x_c - A_{22}^{-1} B_2 u$$

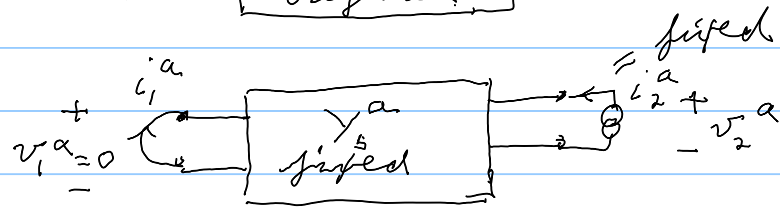
5) → 3) $$\dot{x}_c = \left[A_{11} - A_{12} A_{22}^{-1} A_{21}\right] x_c + \left[B_1 - A_{22}^{-1} B_2\right] u$$

5) → 2) $$y = \left[C_1 - C_2 A_{22}^{-1} A_{21}\right] x_c - C_2 A_{22}^{-1} B_2 u$$

these are state variable equations, x_c = state

Sensitivity via the adjoint circuit





superscript a = adjoint

$$\mp i_1 \cdot v_1^a + v_1 \cdot i_1^a \mp i_2 v_2^a + v_2 \cdot i_2^a \mp i_b^T \cdot v_b^a + i_b^{aT} \cdot v_b = 0$$

changes on this, use δ to designate

$$\underbrace{\delta(i_1 v_1^a)}_{0} + \underbrace{\delta(v_1 \cdot i_1^a)}_{v_1 \cdot \delta i_1^a \,=\, 0} \pm \underbrace{\delta(i_2 v_2^a)}_{0} \pm \delta(i_b^T \cdot v_b^a) + \delta(i_b^{aT} \cdot v_b) + \delta(v_2 \cdot i_2^a) = 0$$

$$\pm \delta i_b^T \cdot v_b^a + i_b^{aT} \cdot \delta v_b + i_2^a \cdot \delta v_2$$

$i_b = Y_b v_b$, $\quad i_b^a = Y_b^a v_b^a$

$\Rightarrow \delta i_b = \delta Y_b v_b + Y_b \delta v_b$, $\quad \delta i_b^a = \delta Y_b^a v_b^a + Y_b^a \delta v_b^a \equiv 0$ no change in adjoint circuit

$$i_2^a \cdot \delta v_2 = \mp \delta i_b^T \cdot v_b^a - i_b^{aT} \cdot \delta v_b = \mp \left[v_b^T \cdot \delta Y_b^T + \delta v_b^T Y_b^T \right] v_b^a - \left[v_b^{aT} Y_b^{aT} \right] \delta v_b$$

Transpose = same result

$$i_2^a \delta v_2 = \mp v_b^{aT} \left[\delta Y_b v_b + Y_b \delta v_b \right] - v_b^{aT} Y_b^{aT} \delta v_b$$

$$= + v_b^{aT} \left[\delta Y_b \right] v_b + v_b^{aT} \left[\underbrace{Y_b \delta v_b - Y_b^{aT} \delta v_b}_{\text{choose adjoint circuit } Y_b^{aT} = Y_b} \right]$$

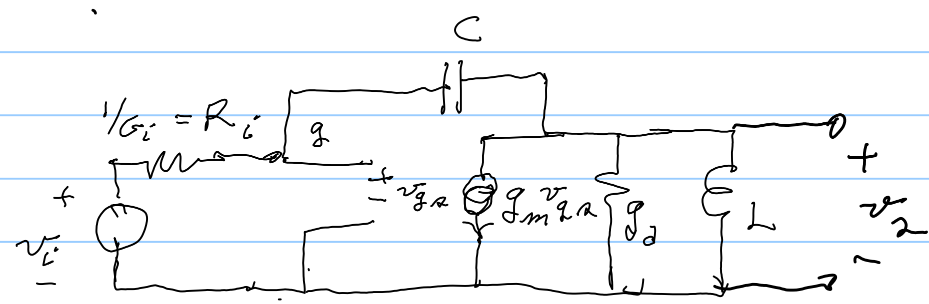
$$\Rightarrow i_2^a \cdot \delta v_2 = + v_b^{aT} \cdot \delta Y_b \cdot v_b$$

if y_{ij} changes then v_2 changes as $\delta v_2 = v_{b_i}^a \cdot \delta y_{ij} \, v_{b_j} / i_2^a$

$$\frac{\delta v_2}{\partial y_{ij}} = \frac{v_{b_i}^a \cdot v_{b_j}}{i_2^a} \Leftarrow$$ no derivative on RH side only circuit analysis

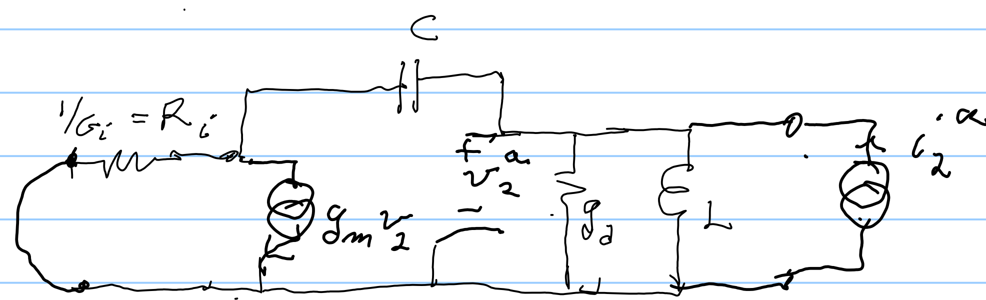
$$S_{y_{ij}}^{v_2/v_1} = \frac{\partial v_2/v_1}{\partial y_{ij}} \Big/ \frac{v_2/v_1}{y_{ij}} \Rightarrow S_x^T = \frac{\partial T/\partial x}{T/x}$$ sensitivity of $T(x)$ w.r.t. x

Ex:



desire

$$\frac{d v_2}{d g_m} = \frac{v_2 v_2^a}{i_2^a}$$



adjoint