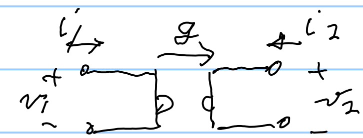
Note Title 9/10/2015

EE610
09/10/15

gyrator



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow i = Y v$$

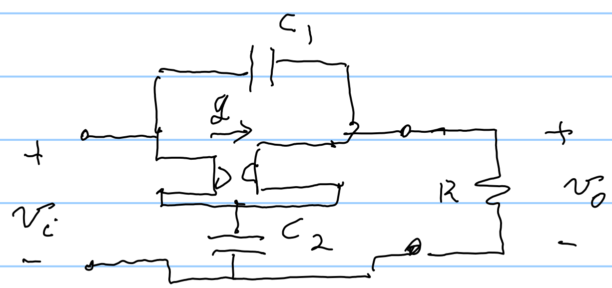
g = gyration conductance

$$i_1 v_1 + i_2 v_2 = P_{in} = -g v_2 v_1 + g v_1 v_2 \equiv 0$$

instantaneously lossless (passive)

$$Z_{gy} = Y^{-1} = \frac{1}{g^2}\begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix}$$

$$T(s) = \frac{V_o}{V_i}(s) = \frac{s^2 + 2\beta_1 \omega_0 s + \omega_0^2}{s^2 + 2\beta_2 \omega_0 s + \omega_0^2}$$

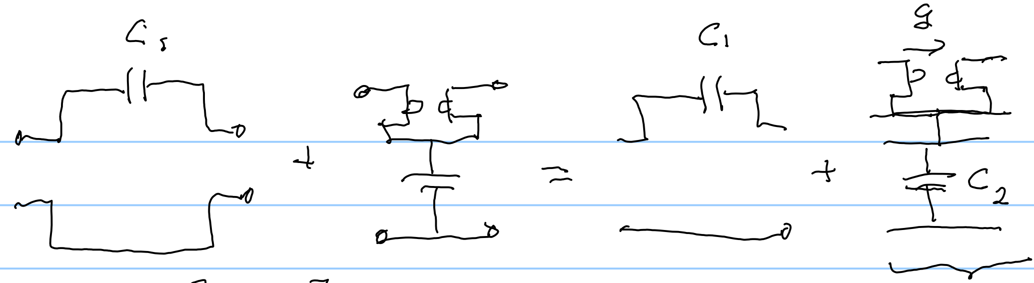


$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2 = -y_L V_2$$

$$y_{21} V_1 = -(y_L + y_{22}) V_2$$

$$\frac{V_2}{V_1} = \frac{-y_{21}}{y_{22} + y_L}$$



$$Z_g = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix}$$

$$Z_{C_2} = \frac{1}{sC_2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$Y_{C_1} = sC_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$Z_2 = Z_g + Z_{C_2} = \begin{bmatrix} 1/sC_2 & r + 1/sC_2 \\ -r + 1/sC_2 & 1/sC_2 \end{bmatrix}$$

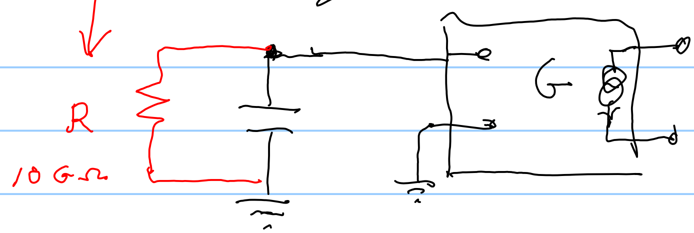
$$Y_2 = Z_2^{-1} = \frac{1}{r^2}\begin{bmatrix} 1/sC_2 & -r - 1/sC_2 \\ r - 1/sC_2 & 1/sC_2 \end{bmatrix} \Rightarrow Y = Y_{C_1} + Y_2 = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$y_{21} = -sC_1 + g - 1/sr^2C_2 , \quad y_{22} = sC_1 + \frac{1}{r^2 sC_2}$$

$$\frac{V_2}{V_1} = \frac{(sC_1 - g + 1/sr^2C_2)}{G + sC_1 + 1/sr^2C_2} = \frac{s^2 - \frac{g}{C_1}s + \frac{1}{r^2C_1C_2}}{s^2 + \frac{G}{C_1}s + 1/r^2C_1C_2} = \frac{s^2 + \beta_1\omega_0 + \omega_0^2}{s^2 + \beta_2\omega_0 + \omega_0^2}$$

convergence R

no DC path to ground



for the above $T(s)$, the state variable equations are: x = state

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \frac{d}{dt} x = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2\beta_2\omega_0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad u = v_i$$

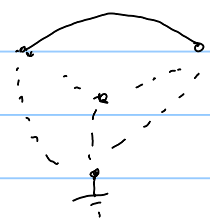
output $= v_o = y = \begin{bmatrix} 0 & 2\omega_0(\beta_1 - \beta_2) \end{bmatrix} x + \begin{bmatrix} 1 \end{bmatrix} u$

want $T(s)$: $\quad sX(s) = AX(s) + BV_i \Rightarrow X(s) = (s1_2 - A)^{-1} B V_i$

$$V_o = CX(s) + DV_i$$

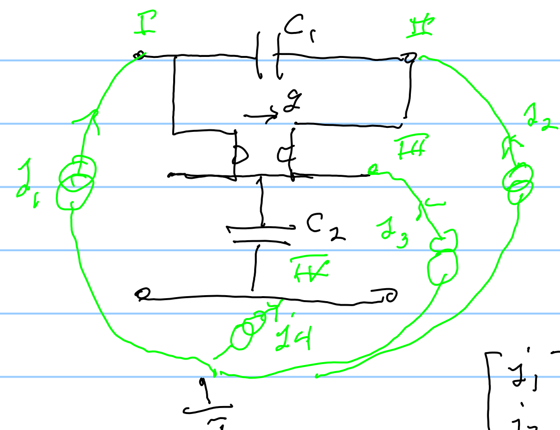
$$V_o = \left[C(s1_2 - A)^{-1}B + D\right] V_i \Longrightarrow T(s) = D + C(s1_2 - A)^{-1}B$$

Indefinite Y_{ind} (all entries in a row sum to 0 & same for columns)



sum all currents at a node to zero including the external current source
and all the currents will sum to zero

Ex:



$$j_1 = sC_1(v_I - v_{II}) - g(v_{II} - v_{III})$$

$$j_2 = sC_1(v_{II} - v_I) + g(v_I - v_{III})$$

$$j_3 = sC_2(v_{III} - v_{IV}) + g(v_{II} - v_{III}) - g(v_I - v_{III})$$

$$j_4 = sC_2(v_{IV} - v_{III})$$

$$\begin{bmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \end{bmatrix} = \begin{bmatrix} sC_1 & -g-sC_1 & g & 0 \\ g-sC_1 & sC_1 & -g & 0 \\ -g & g & sC_2 & -sC_2 \\ 0 & 0 & -sC_2 & sC_2 \end{bmatrix} \begin{bmatrix} v_I \\ v_{II} \\ v_{III} \\ v_{IV} \end{bmatrix}$$

Move $\underline{i}$ to node IV $\Rightarrow v_{IV} = 0$; ignore 4th row

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \cancel{i_4} \end{bmatrix} = \begin{bmatrix} sC_1 & -g-sC_1 & g & \cancel{0} \\ g-sC_1 & sC_1 & -g & \cancel{0} \\ -g & g & sC_2 & \cancel{-sC_2} \\ \cancel{0} & \cancel{0} & \cancel{-sC_2} & \cancel{sC_2} \end{bmatrix} \begin{bmatrix} v_I \\ v_{II} \\ v_{III} \\ \cancel{v_{IV}} \end{bmatrix}$$

next force $i_3 = 0$ as when use as a 2-port no node 3 excitation

$$\begin{bmatrix} i_1 \\ i_2 \\ \hline i_3 = 0 \end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right] \begin{bmatrix} v_I \\ v_{II} \\ v_{III} \end{bmatrix}$$

$$0 = Y_{21} \begin{bmatrix} v_I \\ v_{II} \end{bmatrix} + Y_{22} v_{III}$$

$$0 = [-g \quad g]\, v + sC_2 v_{III}$$

$$v_{III} = Y_{22}^{-1} (-Y_{21})\, v \quad ; \quad i = \left(Y_{11} + Y_{12} (-Y_{22}^{-1}) Y_{21}\right) v$$

$$i = Y_{2\text{-port}}\, v \quad , \quad Y = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}$$

$$= \begin{bmatrix} sC_1 & -g-sC_1 \\ g-sC_1 & sC_1 \end{bmatrix} - \begin{bmatrix} g \\ -g \end{bmatrix} \frac{1}{sC_2} [-g \quad g]$$