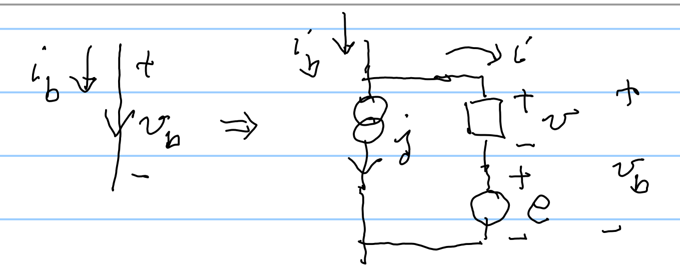
EE 610
09/08/15

laws of devices $\left\{ Av = Bi \right.$



$i_b = i + j$

$v_b = v + e$

j = independent currents, e = independent voltage

laws of connection $\left\{ \text{KVL, KCL} \right.$ $\quad v_b = \mathcal{C}^T v_t$, $i_b = \mathcal{J}^T i_\ell$

$\Uparrow$ tree voltages $\qquad \Uparrow$ link currents

$v = v_b - e$, $i = i_b - j \Rightarrow Av = Av_b - \cancel{Ae} = Bi_b - Bj = Bi$

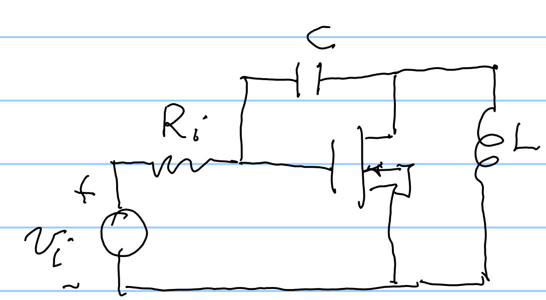
$Av_b - Ae = A\mathcal{C}^T v_t - Ae = B\mathcal{J}^T i_\ell - Bj$

circuit elements $\Rightarrow A\mathcal{C}^T v_t - B\mathcal{J}^T i_\ell = Ae - Bj \Leftarrow$ sources ; $\begin{bmatrix} v_t \\ i_\ell \end{bmatrix} \Leftarrow$ b-vector

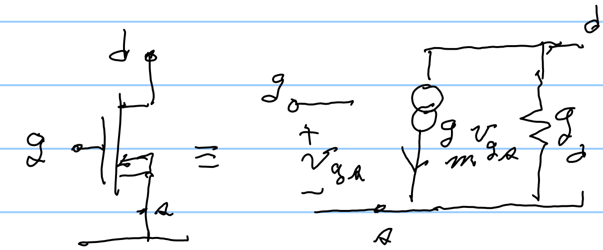
$= \begin{bmatrix} A\mathcal{C}^T & -B\mathcal{J}^T \end{bmatrix} \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = Ae - Bj$

desire $\left[AC^T \,\vdots\, -BJ^T \right]^{-1} (Ae - Bj) = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$

Ex:



small signal



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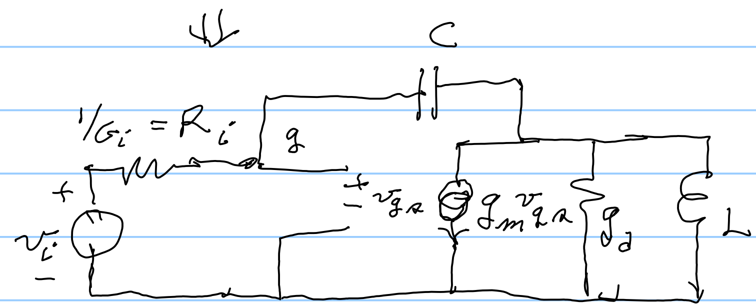
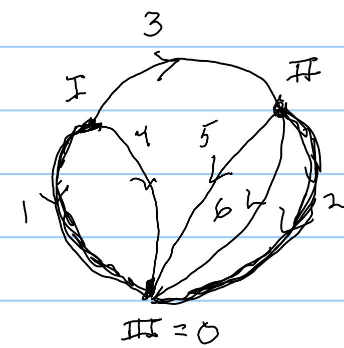
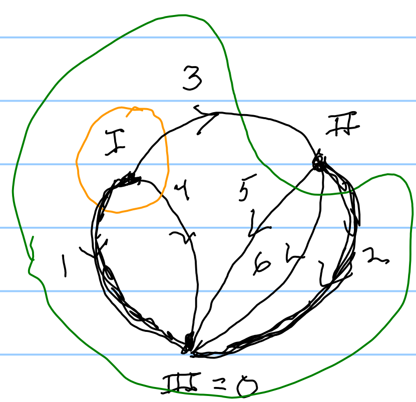




$n = 3$
$b = 6$
$t = 2$
$\ell = 4$

$Av = Bi$, A & B are 6×6

$$\begin{bmatrix} G_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & sC & 0 & 0 & 0 \\ \Delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_d \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & sL & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} ; \; j = 0, \; e = \begin{bmatrix} v_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



$$e \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|cccc} \multicolumn{2}{c|}{\overbrace{\quad}^{1_t}} & & & & \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 1 \end{array} \right] i_b \Rightarrow e^T = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \\ \hline 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{array} \right] = \left[\begin{array}{c} 1_2 \\ \hline -K_v^T \end{array} \right]$$

$$J \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|cccc} & & \multicolumn{4}{c}{\overbrace{\qquad\qquad}^{1_4 = 1_\ell}} \\ -1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{array} \right] v_b \Rightarrow J^T = \left[\begin{array}{cccc} -1 & -1 & 0 & 0 \\ 1 & 0 & -1 & -1 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] = \left[\begin{array}{c} K_v \\ \hline 1_4 \end{array} \right]$$

$$
\begin{bmatrix} G_i & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_d \end{bmatrix}
\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \hline 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}
= AC^T =
\begin{bmatrix} G_i & 0 \\ 0 & 1 \\ \Delta C & -\Delta C \\ 0 & 0 \\ 0 & g_m \\ 0 & g_d \end{bmatrix}
$$

$$
-\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta L & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} -1 & -1 & 0 & 0 \\ 1 & 0 & -1 & -1 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
= -BD^T = -
\begin{bmatrix} -1 & -1 & 0 & 0 \\ \Delta L & 0 & -\Delta L & -\Delta L \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
=
\begin{bmatrix} +1 & 1 & 0 & 0 \\ -\Delta L & 0 & \Delta L & \Delta L \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}
$$

$$\left[\begin{array}{cc|cccc} G_i & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -sL & 0 & sL & sL \\ sC & -sC & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & g_m & 0 & 0 & -1 & 0 \\ 0 & g_d & 0 & 0 & 0 & -1 \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} G_i v_i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Leftarrow$ circuit equations

can drop row & column 4

If the inductor term is sent to the A matrix then

$$A v = i \Rightarrow A = Y_{b\times b}$$

$$\left[Y_{b\times b} e^T \,\middle|\, -\mathcal{J}^T \right] \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = \underbrace{Y_{bb} \times \mathcal{E}}_{\text{Norton equivalent current sources}}$$

as $P_{in} = 0$; $v_b^T i_b = v_t^T e \mathcal{J}^T i_\ell = 0 \Rightarrow e \mathcal{J}^T = 0_{t\times \ell}$

$$\mathcal{E} \left[Y_{b\times b} e^T \,\middle|\, -\mathcal{J}^T \right] \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = \left[e Y_{b\times b} e^T \,\middle|\, 0_{t\times \ell} \right] \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = e Y_{bb} \mathcal{E}$$

$$e\,Y_{bb}\,e^{T} v_t = e\,Y_{bb}\,e \;\Rightarrow\; v_t = \left[e\,Y_{b\times b}\,e^{T}\right]^{-1}\left[e\,Y_{b\times b}\,e\right]$$