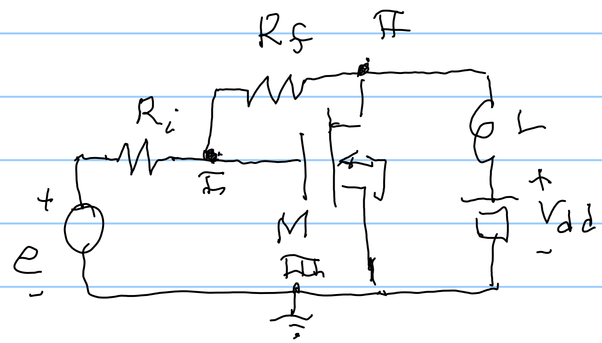
EE 610
09/01/15

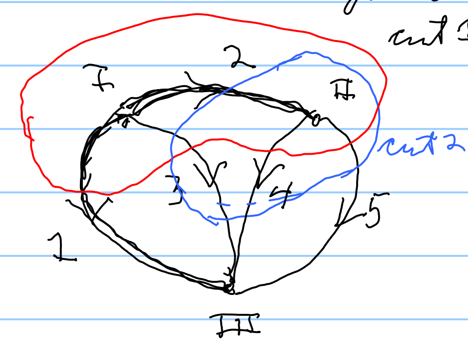
Note Title 9/1/2015

KCL $\Rightarrow$ Σ currents into a sphere $= 0$

KVL $\Rightarrow$ Σ voltages around a closed path $= 0$



$\Longrightarrow$



$n = 3$ nodes

$b = 5$ branches

$t = 2$ tree

$l = 3$ links

$b = t + l$

$$v_b = \begin{bmatrix} v_1 \\ v_2 \\ \hline v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} v_t \\ v_l \end{bmatrix}, \quad i_b = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} i_t \\ i_l \end{bmatrix}$$

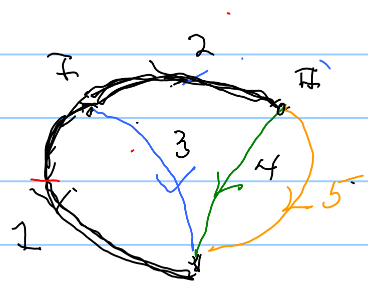
KCL

$$0 = 1 \cdot i_1 + 0 \cdot i_2 + 1 \cdot i_3 + 1 \cdot i_4 + 1 \cdot i_5$$

$$0 = 0 \cdot i_1 + 1 \cdot i_2 + 0 \cdot i_3 - 1 \cdot i_4 - i_5$$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & -1 \end{bmatrix} i_b \Rightarrow \underset{\sim}{0} = e\, i_b$$

KVL



$$0 = -1.v_1 + 0.v_2 + 1.v_3 + 0.v_4 + 0.v_5$$

$$0 = -1.v_1 + 1.v_2 + 0.v_3 + 1.v_4 + 0.v_5$$

$$0 = -1 v_1 + 1 v_2 + 0.v_3 + 0.v_4 + 1.v_5$$

tie set eqs.

$$\underline{0} = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix} v_b \Rightarrow \underline{0} = \mathcal{T}.v_b$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{= 1_3}$$

Know Pin from outside $= 0$

$$i_b^T \cdot v_b = [i_1\ i_2\ i_3\ i_4\ i_5] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = 0$$

$$KVL \Rightarrow \underline{0} = \mathcal{T} \cdot \begin{bmatrix} v_t \\ v_\ell \end{bmatrix} = [K_v \ \vdots \ 1_\ell] v_b \Rightarrow v_\ell = -K_v \cdot v_t \Rightarrow v_b = \begin{bmatrix} v_t \\ v_\ell \end{bmatrix} = \begin{bmatrix} 1_t \\ \hline -K_v \end{bmatrix} v_t$$

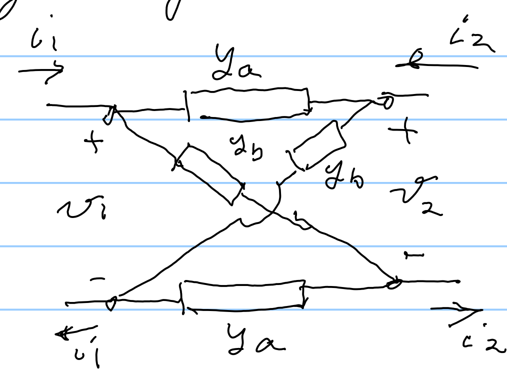
$$KCL \Rightarrow \underline{0} = \mathcal{C} \begin{bmatrix} i_t \\ i_\ell \end{bmatrix} = [1_t \ \vdots \ K_i] \begin{bmatrix} i_t \\ i_\ell \end{bmatrix} = \underline{0} \qquad i_t = -K_i \cdot i_\ell \Rightarrow i_b = \begin{bmatrix} i_t \\ i_\ell \end{bmatrix} = \begin{bmatrix} -K_i \\ 1_\ell \end{bmatrix} i_\ell$$

$$i_b^T v_b = i_\ell^T \left[-K_i^T \,\vdots\, 1_\ell\right] \begin{bmatrix} 1_t \\ -K_v \end{bmatrix} v_t = 0 \Rightarrow \left[-K_i^T \,\vdots\, 1_\ell\right] \begin{bmatrix} 1_t \\ -K_v \end{bmatrix} = 0_{\ell \times t}$$

$$\Rightarrow \quad -K_i^T \cdot 1_t - K_v \cdot 1_\ell = 0_{\ell \times t} \Rightarrow -K_i^T = K_v$$

$$\left.\begin{aligned} e &= \left[1_t \,\vdots\, -K_v^T\right] \\ \sigma_i &= \left[K_v \,\vdots\, 1_\ell\right] \end{aligned}\right\} \text{laws of connections for electronic circuits}$$

Y matrix for symmetric lattice



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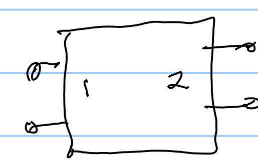
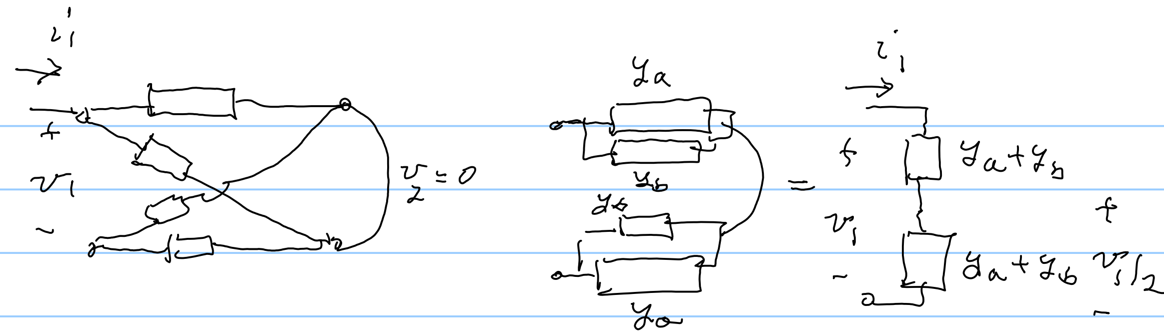


$$i = Yv = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{12} & y_{11} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$$



$$i_1 = (y_a + y_b) \cdot \frac{v_1}{2} \Longrightarrow y_{11} = y_{22} = \frac{1}{2}(y_b + y_a)$$

$$y_{12} = \frac{1}{2}(y_b - y_a) \qquad Y = \frac{1}{2}\begin{bmatrix} y_b + y_a & y_b - y_a \\ y_b - y_a & y_b + y_a \end{bmatrix}$$