1. (60 points, Lattice, Graph, Constant R)


For the above circuit
a) Use the graph on the right and obtain the augmented incidence matrix $\mathrm{I}_{\text {inc_aug. }}$. Then delete the last row to get the $3 \times 6$ incidence matrix $\mathrm{I}_{\mathrm{inc}}$.
b) Next form the $\operatorname{det}\left(\mathrm{I}_{\mathrm{inc}} \mathrm{I}_{\mathrm{inc}}{ }^{\mathrm{T}}\right.$ ) which gives the number of possible trees. (you may wish to locate a proof of this fact in the literature especially since the result stated in the book [using $\mathrm{Aa}=\mathrm{I}_{\text {inc_aug }}$ instead of $\mathrm{A}=\mathrm{I}_{\mathrm{inc}}$ ] is in error).
c) Chose the tree to be the first numbered branches and give the cut-set and tie-set matrices.
d) The lattice has the admittance matrix

$$
\mathrm{Y}=\frac{1}{2}\left[\begin{array}{cc}
\mathrm{y}_{\mathrm{b}}+\mathrm{y}_{\mathrm{a}} & \mathrm{y}_{\mathrm{b}}-\mathrm{y}_{\mathrm{a}} \\
\mathrm{y}_{\mathrm{b}}-\mathrm{y}_{\mathrm{a}} & \mathrm{y}_{\mathrm{b}}+\mathrm{y}_{\mathrm{a}}
\end{array}\right]
$$

and when loaded in $y_{L}$ (in this case $G=1 / R$ ) has the input admittance

$$
y_{i}=y_{11}-y_{12}\left(y_{22}+y_{L}\right)^{-1} y_{21}
$$

Find $y_{i}$ for the above lattice and show that $y_{i}=G$ is obtained when $y_{a} y_{b}=G^{2}$.
e) Chose $y_{a}$ to go with a capacitor GC; give $y_{a}$ and $y_{b}$ and draw the resulting lattice.
f) For the constant $R$ lattice of d) placed in the above circuit, give the voltage transfer function $\mathrm{v}_{\mathrm{o}}(\mathrm{s}) / \mathrm{e}(\mathrm{s})$
2. (40 points, cascade of constant $R$ lattices)


For this circuit use different capacitors for the two lattices and find the resulting voltage transfer function, which should be all-pass. Then choose $\mathrm{R}=50 \mathrm{Ohms}$ and $\mathrm{C}_{2}=2 \mathrm{C} 1=2 \mathrm{nFd}$, and set up the circuit in Spice to do a frequency response over 10 Hz to 1 GHz . Comment on the result.

Additional problems, not for grading

1. The following 10 node 15 edge graph, when undirected, is known as the Petersen Graph and, due to each node having three branches, is of interest in the four color problem. Choose the numbering given (1 through 10 for nodes and I through XV for edges)
a. Choose the tree formed by the thicker edges (I through IX) and give the cut-set and tie-set matrices using the orientations given.
b. Determine the number of trees (quite a large number).

