ED 610 Final Exam Fall 2015
Open Book Open Notes 150 points, 2 hours.
Notebooks are due at the end of the exam. Good luck and have a good semester break.

1. (60 points, 40 minutes)


For the above circuit use the node numbering given
a) Unground node 4 and give the indefinite ( $4 \times 4$ ) indefinite matrix Mind
b) Ground node four to give the definite ( $3 \times 3$ ) matrix Ydef
c) Eliminate the internal node 3 to get the 2-port matrix Y
d) Explain why $Y$ only exists if $g_{3}$ is non-zero
2. (60 points, 40 minutes)

Consider the input admittance $y(s)=\left[\left(s^{2}+1\right)\left(s^{2}+9\right)\right] /\left[s\left(s^{2}+a\right)\right]$
a) For what values of a is this a lossless PR function?
b) Choose $\mathrm{a}=4$ (for which $\mathrm{y}(\mathrm{s})$ is PR lossless) and give a $2^{\text {nd }}$ Foster synthesis.
c) For your synthesis feed by a current source and draw the resulting graph having a branch for every circuit component. Indicate a tree with as many branches as possible connected to the bottom of the current source; direct the tree branches downward and number them sequentially with lowest numbers from left to right (direct links left to right or down and numbered sequentially after the tree branches). Give the resulting cut set and tie set matrices.
3. (30 points, 30 minutes)

A 2-port is described by the following time-varying admittance where all functions are continuously differentiable. $s$ is the derivative operator $s=d(.) / d t$ with $\mathrm{s}[\mathrm{f}(\mathrm{t}) \mathrm{v}(\mathrm{t})]=\mathrm{f}(\mathrm{t}) \mathrm{s}[\mathrm{v}(\mathrm{t})]+[\mathrm{df}(\mathrm{t}) / \mathrm{dt}] \mathrm{v}(\mathrm{t})$; thus, $\mathrm{sc}(\mathrm{t})=\mathrm{c}(\mathrm{t}) \mathrm{s}+\mathrm{dc}(\mathrm{t}) / \mathrm{dt}$ is an operator in the following matrix.

$$
\mathrm{Y}(\mathrm{~s}, \mathrm{t})=\left[\begin{array}{cc}
0 & \mathrm{~g}_{1}(\mathrm{t}) \\
-\mathrm{g}_{2}(\mathrm{t}) & \mathrm{sc}(\mathrm{t})
\end{array}\right]
$$

a) The adjoint of a matrix $\mathrm{Y}(\mathrm{s}, \mathrm{t})=\mathrm{sC}(\mathrm{t})+\mathrm{G}(\mathrm{t})=\mathrm{C}(\mathrm{t}) \mathrm{s}+\mathrm{dC}(\mathrm{t}) / \mathrm{dt}+\mathrm{G}(\mathrm{t})$ is $Y^{\mathrm{a}}(\mathrm{s}, \mathrm{t})=-\mathrm{C}(\mathrm{t})^{\mathrm{T}} \mathrm{s}+\mathrm{G}(\mathrm{t})^{\mathrm{T}}$ with ${ }^{\mathrm{T}}$ the transpose. Assume that the lossless condition is $\mathrm{Y}+\mathrm{Y}^{\mathrm{a}}=0$ [=the zero matrix]. When is the circuit for the above Y lossless?
b) If $\mathrm{c}=0$ give the conditions for the 2-port to be passive.

