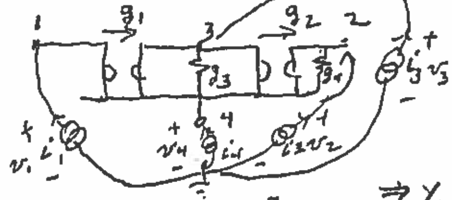
EE 610 F2015 Final Solution

#1.



$i_1 = -g_1(v_3 - v_4)$; $i_2 = g_2(v_3 - v_4) + g_4(v_2 - v_4)$

$i_3 = g_3(v_3 - v_4) + g_1(v_1 - v_4) + (-g_2)(v_2 - v_4) \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = Y_{ind} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$

$i_4 = -(i_1 + i_2 + i_3)$

a) $\left\{ \begin{array}{l} \text{use } Y_{gyr} = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} \Rightarrow \\ \text{if use } \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \text{ replace } g\text{'s by } -g\text{'s} \end{array} \right\}$ $\Rightarrow Y_{ind} = \begin{bmatrix} 0 & 0 & -g_1 & +g_1 \\ 0 & g_4 & g_2 & -g_2 - g_4 \\ +g_1 & -g_2 & g_3 & -g_3 - g_1 + g_2 \\ -g_1 & g_2 - g_4 & g_1 - g_2 - g_3 & g_3 + g_4 \end{bmatrix}$

b) ground 4 $\Rightarrow v_4 = 0$ & ignore i_4 $\Rightarrow$ delete row & column 4 $\Rightarrow Y_{def} = \begin{bmatrix} 0 & 0 & -g_1 \\ 0 & g_4 & g_2 \\ g_1 & -g_2 & g_3 \end{bmatrix}$

c) eliminate 3 $\Rightarrow i_3 = 0 \Rightarrow$ last row of $Y_{def} = 0$

$0 = g_1 v_1 - g_2 v_2 + g_3 v_3 \Rightarrow v_3 = -\frac{1}{g_3}[g_1 v_1 - g_2 v_2]$

$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & g_4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} -g_1 \\ g_2 \end{bmatrix} v_3 \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 + \frac{g_1^2}{g_3} v_1 - \frac{g_1 g_2}{g_3} v_2 \\ 0 - \frac{g_1 g_2}{g_3} + \frac{g_2^2}{g_3} v_2 + g_4 \end{bmatrix}$

$\Rightarrow Y = \frac{1}{g_3} \begin{bmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & +g_2^2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & g_4 \end{bmatrix}$

d) If $g_3 = 0 \Rightarrow$ an open circuit & the 2-port becomes an ideal transformer which has no Y. Note if $g_4 = 0$ row 2 of Y is row $1 \times -\frac{g_2}{g_1} \Rightarrow i_2 = -\frac{g_2}{g_1} i_1$ & if $g_3 = 0 \Rightarrow i_3 = 0 \Rightarrow g_1 v_1 = g_2 v_2$

$\Rightarrow v_2 = \frac{g_1}{g_2} v_1$ & $i_1 = -\frac{g_2}{g_1} i_2 \Rightarrow$ transformer, $T = g_1/g_2$ = turns ratio

#2. $y(s)$ PR requires $a \geq 0$ for no RHP poles. But $a \neq 0$ as otherwise a triple pole @ 0.

$\therefore$ $a > 0$ & all poles & zeros are on $j\omega$ axis where they must alternate

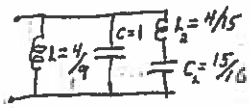
a) So $1 < a < 9$ & $Y(s)$ is lossless

b) @ $a = 4$ $\quad y(s) = \frac{(s^2+1)(s^2+9)}{s(s^2+4)} = \frac{k_0}{s} + k_\infty s + \frac{2k_2 s}{s^2+4}$ for 2nd Foster: $k_0 = s y(s)\big|_{s=0} = \frac{1 \times 9}{4} = 9/4$

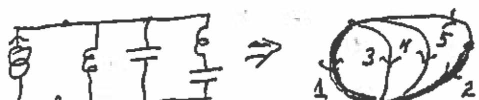
$k_\infty s = s^4/s^3 \Rightarrow k_\infty = 1$;

$2k_2 = \frac{s^2+4}{s} y(s)\Big|_{s^2=-4} = \frac{(-3)(5)}{-4} = 15/4$

$\Rightarrow y(s) = \frac{9/4}{s} + s + \frac{\frac{15}{4} s}{s^2+4} \Rightarrow$



c)



where dark branches = tree

Cut set: KCL for 1 & 2 $\Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 \end{array}\right] i_b \Rightarrow C = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix} = [1_2 \mid K]$

Tie set: KVL for 3, 4, 5 $\quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|ccc} -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \end{array}\right] v_b \Rightarrow T = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix} = [-K^T \mid 1_3]$

as a check

#3. $Y(s,t) = \begin{bmatrix} 0 & g_1(t) \\ -g_2(t) & sc(t) \end{bmatrix}$, $Y^a(s,t) = \begin{bmatrix} 0 & -g_2(t) \\ g_1(t) & -c(t)s \end{bmatrix}$

$Y + Y^a = \begin{bmatrix} 0 & -g_2(t)+g_1(t) \\ -g_2(t)+g_1(t) & sc(t) - c(t)s \end{bmatrix}$ but $sc(t) = c(t)s + dc/dt = c(t)s + \dot{c}(t)$

$= \begin{bmatrix} 0 & -g_2+g_1 \\ -g_2+g_1 & c(t)s + \dot{c} - c(t)s \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \underline{g_2(t) = g_1(t),\ \dot{c} = 0 \Rightarrow c(t) = \text{const.}}$

lossless condition

a) $\therefore Y(s,t) = \begin{bmatrix} 0 & g(t) \\ -g(t) & sC \end{bmatrix}$, C = constant is lossless

b) To be passive $\mathcal{E}(t) = \int_{-\infty}^{t} v^T(\tau)\, i(\tau)\, d\tau \geq 0$

Here when $c = 0 \Rightarrow Y(s,t) = \begin{bmatrix} 0 & g_1(t) \\ -g_2(t) & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = Y(s,t) \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$

$v^T(\tau)\, i(\tau) = [v_1 \quad v_2] \begin{bmatrix} 0 & g_1(t) \\ -g_2(t) & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = [-v_2 g_2 \quad v_1 g_1] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$= [-v_1 v_2 g_2 + v_1 v_2 g_1] = v_1 v_2 [g_1 - g_2](t)$

as $v_1 v_2$ can be any sign choose it opposite to sign of $g_1 - g_2$

& then $\mathcal{E}(t) < 0 \Rightarrow \underline{g_1(t) - g_2(t) \equiv 0 \text{ for all } t} \Rightarrow \mathcal{E}(t) \equiv 0$

and $Y(s,t) = \begin{bmatrix} 0 & g_1(t) \\ -g_1(t) & 0 \end{bmatrix}$ is passive for "all" functions $g_1(t)$