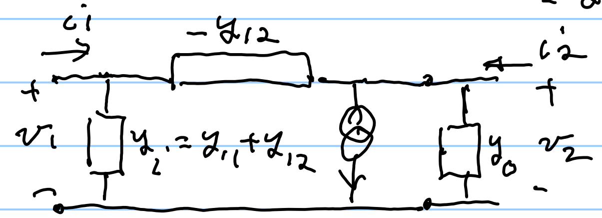
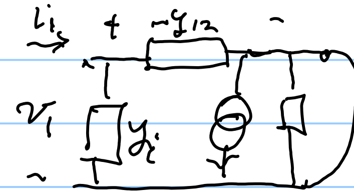
Hybrid-π ⇒ π

$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \Rightarrow i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = Yv$



( ) v_1

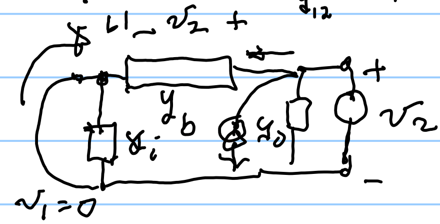
$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$



$y_{11} = y_i - y_{12}$

$y_i = y_{11} + y_{12}$

$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0}$

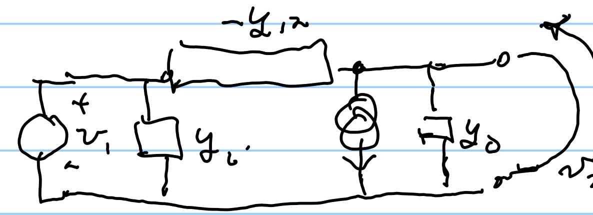


$i_{y_{12}} = -i_1$

$i_{y_{12}} = y_b v_2 = -i_1$

$\left.\frac{i_1}{v_2}\right|_{v_1=0} = -y_b = y_{12}$

$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$



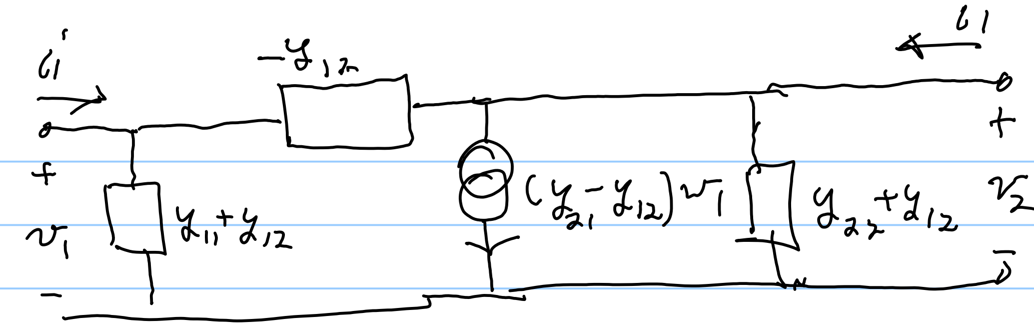
$y_m v_1$

$i_2| = g_m v_1 - (-y_{12}) v_1 = y_{21} v_1$

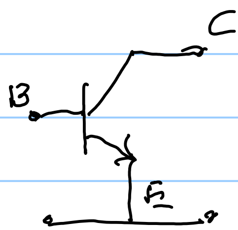
$g_m = y_{21} - y_{12} = g_m$

$y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = y_{22} + y_{12}$

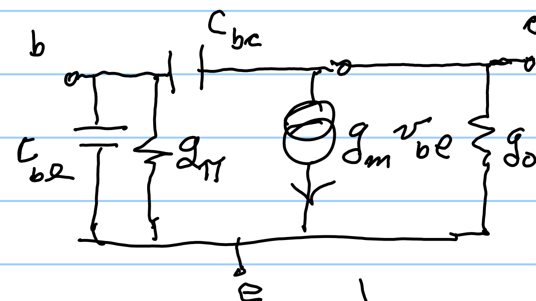
$$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \Rightarrow$$



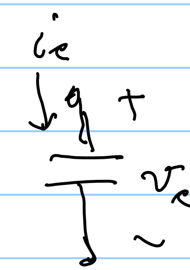
BJT



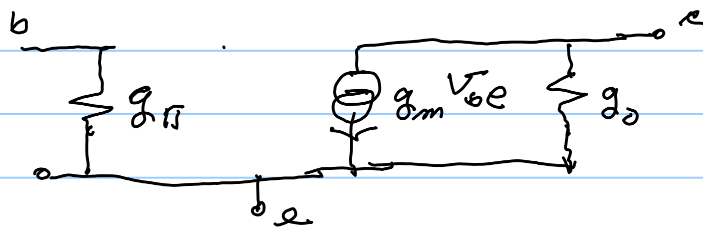
npn



$y_c = sC$



at DC



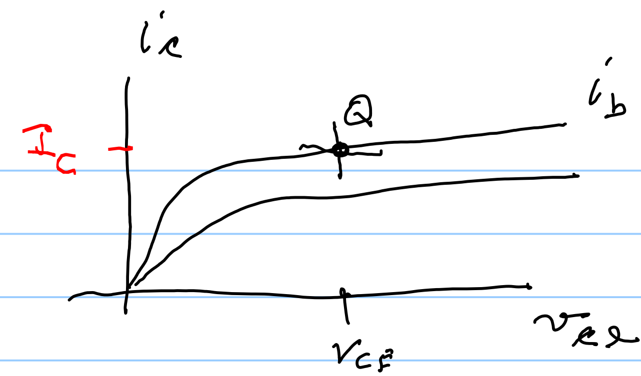
$$Y = \begin{bmatrix} sC_{be} + g_\pi + sC_{bc} & -sC_{bc} \\ g_m - sC_{be} & g_o + sC_{bc} \end{bmatrix}$$

$g_m = y_{21} - y_{12} \Rightarrow y_{21} = g_m + y_{12}$

$y_i = sC_{be} + g_\pi = y_{11} + y_{12}$

$y_{11} = y_i - y_{12} = sC_{be} + g_\pi - (-sC_{bc})$

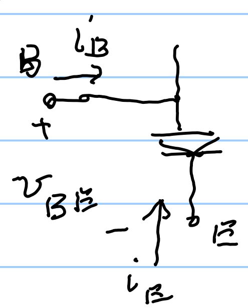
to get Y for the npn

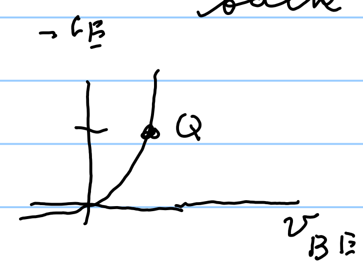


$$\left.\frac{\partial i_c}{\partial i_b}\right|_{v_{ce} = V_{CE}} = \beta$$

This Q has npn in active operation with b-e forward bias & b-c back biased

for g_π





$$-\frac{I_E}{V_T} = g_e \qquad \alpha = \frac{\beta}{1+\beta}$$

$$i_B + i_C + i_E = 0 \equiv i_b + i_c + i_e = 0 \qquad i_B = -i_C - i_E = +\alpha i_E - i_E = (\alpha - 1)\, i_E$$

$$i_C = \beta i_B$$

deduce for g_π i_B vs v_{be} know $g_e = \frac{-i_e}{v_{be}} = \frac{i_B}{(1-\alpha)} \cdot \frac{1}{v_{be}}$; $\frac{i_b}{v_{be}} = g_\pi = (1-\alpha)\, g_e$

$$g_m = \left.\frac{i_c}{v_{be}}\right|_Q = \frac{i_c}{i_b}\cdot\frac{i_b}{v_{be}} = \beta \cdot g_\pi \simeq \beta(1-\alpha)g_e = [\beta(1-\alpha)]\cdot\left(-\frac{I_E}{V_T}\right) \simeq -$$

$$= [\beta(1-\alpha)]\frac{1}{\alpha} I_C/V_T = \frac{\beta}{\beta}\cdot\frac{I_C}{V_T}$$

$$= \frac{I_C}{V_T}, \quad g_\pi = \frac{g_m}{\beta} = \frac{I_C}{\beta V_T}$$

V_A = Early voltage