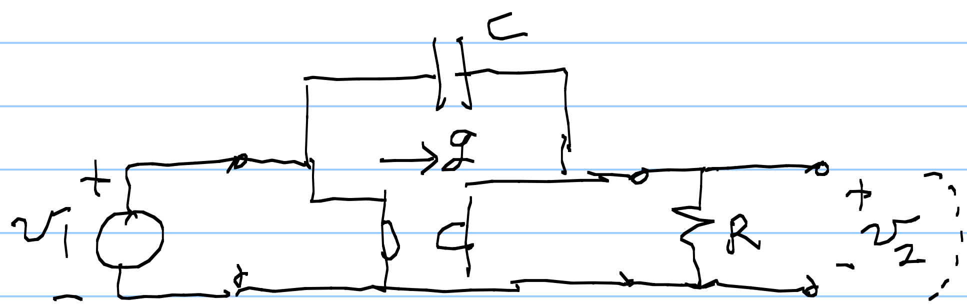
Note Title

EE 610
10/30/14

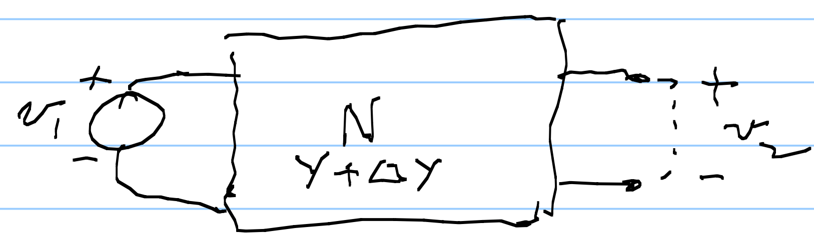
10/30/2014

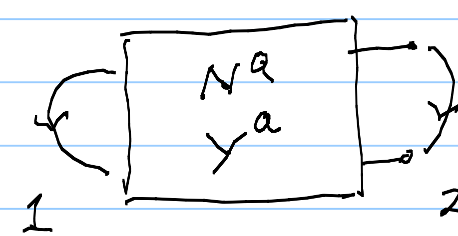
adjoint circuit $Y(\alpha)^a = Y^T(\alpha)$

$$S_x^{T(\alpha)} = \frac{dT(\alpha)/dx}{T(\alpha)/x}$$

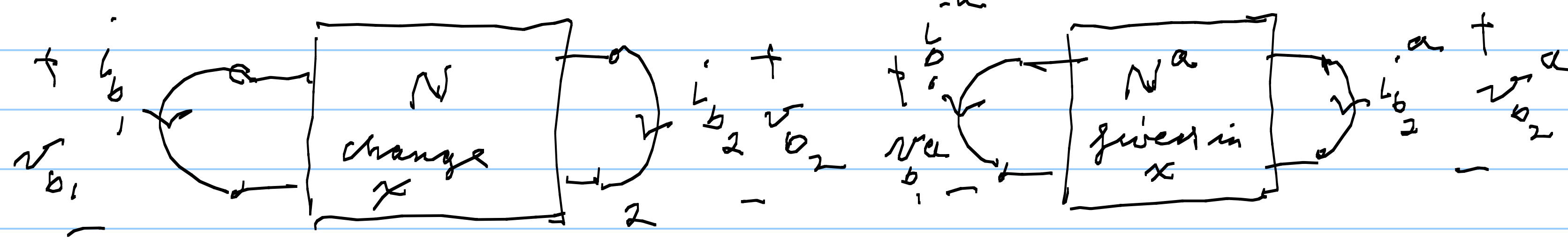


$T(\alpha) = \frac{v_2}{v_1}$





$\Downarrow$



assume N & N^a have the same graph

$i_b^T v_b = 0$ $\qquad$ $i_b^{aT} v_b^a = 0$

$$i_b = \begin{bmatrix} i_{b_1} \\ i_{b_2} \\ \hline i_B \end{bmatrix} \quad v_b = \begin{bmatrix} v_{b_1} \\ v_{b_2} \\ \hline v_B \end{bmatrix}, \quad i_b^a = \begin{bmatrix} i^a_{b_1} \\ i^a_{b_2} \\ i^a_B \end{bmatrix}, \quad v_b^a = \begin{bmatrix} v^a_{b_1} \\ v^a_{b_2} \\ v_B \end{bmatrix}$$

internal $\rightarrow$ branches

$\Rightarrow \quad i_b^{aT} v_b = 0 \;, \quad i_b^T v_b^a = 0$

$\Rightarrow \quad i_b^T v_b^a - i_b^a v_b = 0$

$$[\dot{i}_{b_1} \quad \dot{i}_{b_2} \quad \dot{i}_B]^T \begin{bmatrix} v_{b_1}^a \\ v_{b_2}^a \\ v_B^a \end{bmatrix} - [\dot{i}_{b_1}^a \quad \dot{i}_{b_2}^a \quad \dot{i}_B^a]^T \begin{bmatrix} v_{b_1} \\ v_{b_2} \\ v_B \end{bmatrix} = 0$$

$$\dot{i}_{b_1} v_{b_1}^a + \dot{i}_{b_2} v_{b_2}^a + \dot{i}_B^T \cdot v_B^a - \dot{i}_{b_1}^a v_{b_1} - \dot{i}_{b_2}^a v_{b_2} - \dot{i}_B^{aT} v_B = 0$$

$$0 = \frac{d\dot{i}_{b_1}}{dx} \cdot v_{b_1}^a + \dot{i}_{b_1} \cdot \frac{dv_{b_1}^a}{dx} + \frac{d\dot{i}_B^T}{dx} \cdot v_B^a + \dot{i}_B \frac{dv_B^a}{dx} + \frac{d\dot{i}_{b_2}}{dx} \cdot v_{b_2}^a + \dot{i}_{b_2} \frac{dv_{b_2}^a}{dx^2}$$

$$- \frac{d\dot{i}_{b_1}^a}{dx} \cdot v_{b_1} - \dot{i}_{b_1}^a \frac{dv_{b_1}}{dx} - \frac{d\dot{i}_{b_2}^a}{dx} v_{b_1} - \dot{i}_{b_2}^a \frac{dv_{b_2}}{dx} - \frac{d\dot{i}_B^{aT}}{dx} v_B - \dot{i}_B^{aT} \frac{dv_B}{dx}$$

$$\Rightarrow \frac{d\dot{i}_{b_1}}{dx} \cdot v_{b_1}^a + 0 + \frac{d\dot{i}_B^T}{dx} v_B^a + 0 + \frac{d\dot{i}_{b_2}}{dx} \cdot v_{b_2}^a + 0$$

as $dv_b^a/dx = 0$ as no changes in V^a wrt x

$$-0 - i_{b_1}^{a}\frac{dv_{b_1}}{dx} - 0 - i_{b_2}^{a}\cdot\frac{dv_{b_2}}{dx} - 0 - i_B^{aT}\frac{dv_B}{dx} = 0$$

$\overset{\shortparallel}{0}$ = as v_{b_1} is assumed independent of x

$$\frac{di_{b_1}}{dx}\cdot v_{b_1}^{a} + \underbrace{\frac{di_B^{T}}{dx}\cdot v_B^{a}}_{\#1} + \underbrace{\frac{di_{b_2}}{dx}\cdot v_{b_2}^{a} - i_{b_2}^{a}\frac{dv_{b_2}}{dx}}_{\#3} - \underbrace{i_B^{aT}\frac{dv_B}{dx}}_{\#2} = 0$$

↑

set = 0
by a short
at port 1 of
N^{a}

$$\frac{di_B^{T}}{dx} \Rightarrow i_B = Y_B \cdot v_B$$

$$\frac{di_B^{T}}{dx} = \frac{dv_B^{T}}{dx}\cdot Y_B^{T} + v_B^{T}\frac{dY_B^{T}}{dx}$$

$$i_B^{aT}\cdot\frac{dv_B}{dx} = v_B^{a}\cdot Y_B^{aT}\cdot\frac{dv_B}{dx}$$

$$\#1 \Rightarrow \frac{d v_B^T}{dx} \cdot \gamma_B^T \cdot v_B^a + v_B^T \frac{d \gamma_B^T}{dx} \cdot v_B^a$$

$$\#2 \Rightarrow -i_B^{'aT} \frac{d v_B}{dx} = -v_B^{aT} \gamma_B^{aT} \cdot \frac{d v_B}{dx} = -\left(v_B^{aT} \gamma_B^{aT} \frac{d v_B}{dx} \right)^T$$

$$= -\frac{d v_B^T}{dx} \gamma_B^a v_B^a$$

$$\#_1 + \#_2 \Rightarrow \frac{d v_B^T}{dx} \gamma_B^T v_B^a + v_B^T \frac{d \gamma_B^T}{dx} v_B^a - \frac{d v_B^T}{dx} \gamma_B^a v_B^a$$

force =

by $\gamma_B^T = \gamma_B^a \Rightarrow \gamma_B^a = \gamma_B^T$

from $\#_3$

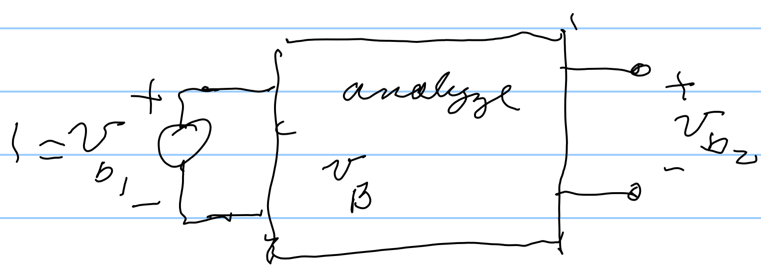
$$i_{b_2}^{'a} \cdot \frac{d v_{b_2}}{dx} = \frac{d i_{b_2}'}{dx} \cdot v_{b_2}^a + v_B^{aT} \frac{d \gamma_B}{dx} \cdot v_B$$

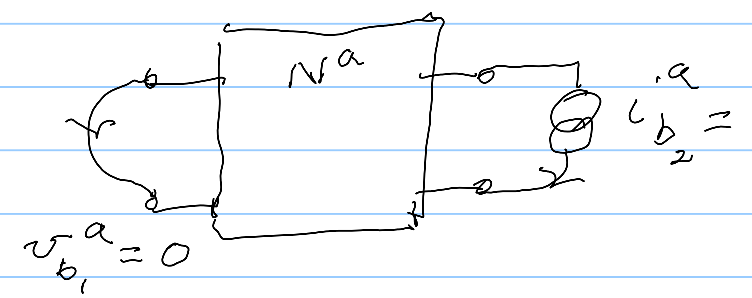
$\Uparrow$

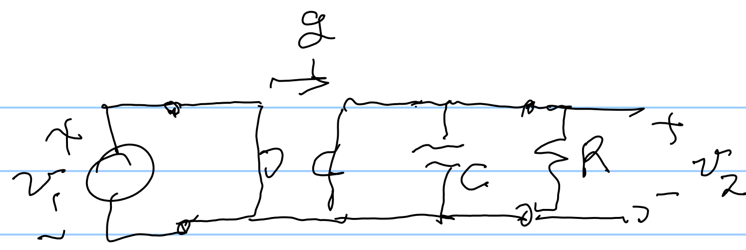
$i_{b_2} = 0$ as an open circuit

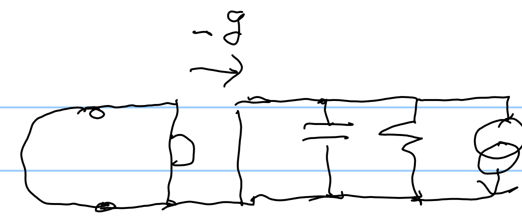
choose $i_{b_2}^a = 1$ $\quad i_{b_2}^a = 1$

$$\frac{d v_{b_2}}{dx} = v_B^{aT} \cdot \frac{d Y_B}{dx} \cdot v_B \quad \Leftarrow \text{a nice result}$$



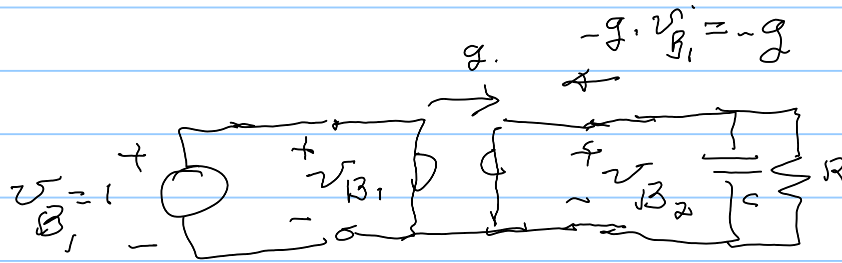


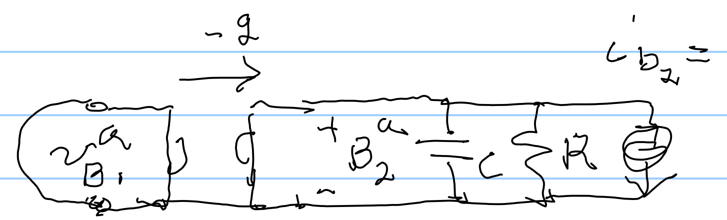


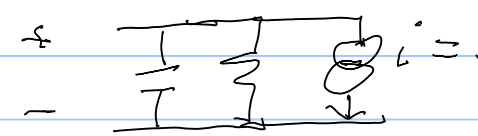


$$S_g^{v_2/v_1}$$

$$Y_B = \begin{bmatrix} 0 & g & 0 & 0 \\ -g & 0 & 0 & 0 \\ 0 & 0 & sC & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \; ; \quad \frac{dY_B}{dg} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$







$$v_{B_1}^{a} = 0, \quad v_{B_2}^{a} = \frac{1}{G + RC}$$

$$\frac{d v_{b_2}}{dg} = \begin{bmatrix} 0 & \frac{1}{G+RC} & x & x \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{b_1} = 1 \\ v_{b_2} = g/G+RC \\ x \\ x \end{bmatrix}$$

$$S_{v}^{T} = \frac{d v_{b2}/dg}{v_{b2}/g}$$