Note Title 10/21/2014

EE610
10/21/14

Scattering matrix

$$2v^i = v + i = v + z_0 i \qquad S v^i = v^r$$
$$2v^r = v - i = v - z_0 i$$

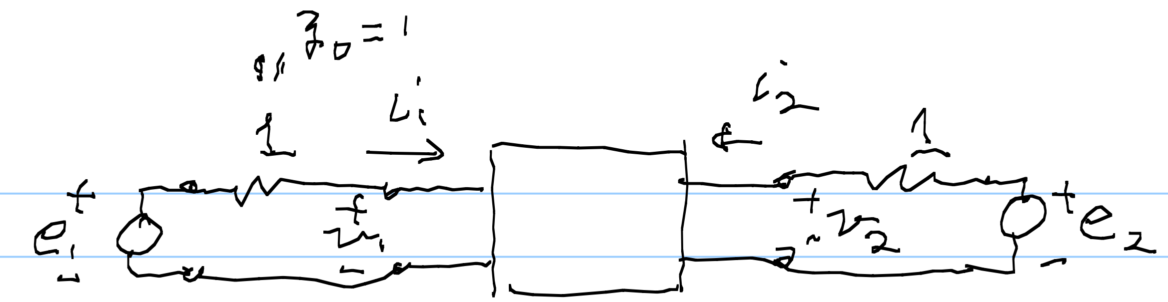
normalize $z_0 = 1$

passive $S \Rightarrow$ bounded real

rational $\Rightarrow$ BR

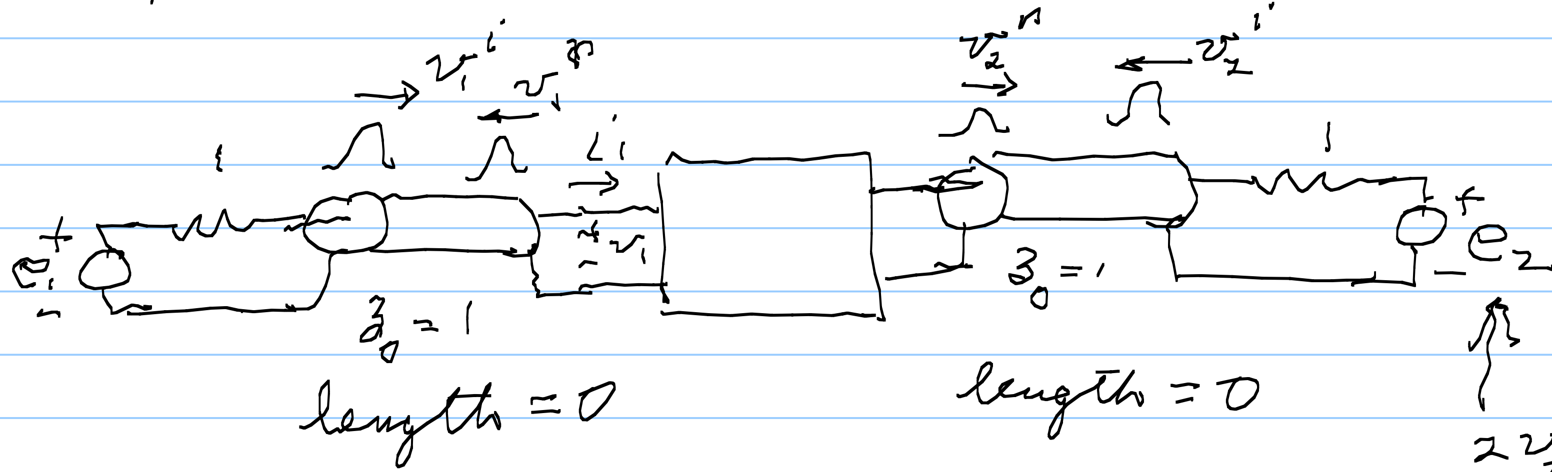
$$2v = 2v^i + 2v^r \Rightarrow \quad v = v^i + v^r = v^i + Sv^i = (1+S)v^i$$
$$2i = 2v^i - 2v^r \qquad\quad i = v^i - v^r = v^i - Sv^i = (1-S)v^i$$

$$Av = Bi \Rightarrow A(1+S)v^i = B(1-S)v^i$$
$$A(v^i + v^r) = B(v^i - v^r) \Rightarrow (A-B)v^i = (A+B)v^r$$
$$S = (A+B)^{-1}(B-A)$$



$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

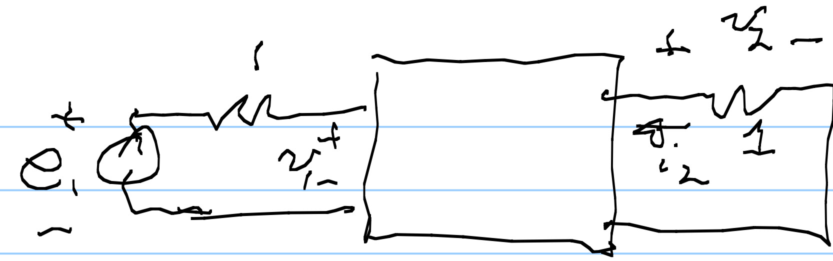
$$e_1 = v_1 + i_1 = 2v_1^i \qquad e_2 = v_2 + i_2 = 2v_2^i$$



$2v_2^i \Rightarrow 0$

if $e_2 = 0 \Rightarrow 2v_2^i = v_2 + i_2 = e_2 \Rightarrow v_2 = -i_2$

$$2v_2^r = v_2 - i_2 = 2v_2, \quad \left.\frac{v_2^r}{v_1^i}\right|_{v_2^i = 0} = \frac{v_2}{e_1/2} = 2\frac{v_2}{e_1} = S_{21}$$



$$\frac{v_2}{e_1} = \frac{1}{2} S_{21}$$

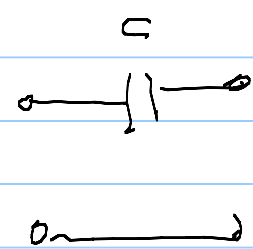
$$S = (B+A)^{-1}(B-A)$$ if Y exists $YV = I$, $Y = A$, $I = 1_n$

$$= (1_n + Y)^{-1}(1_n - Y)$$

if Z exists $1_n V = Z \cdot I$, $1_n = A$, $Z = B$

$$= (Z + 1_n)^{-1}(Z - 1_n)$$

Ex:



$$Y = sC \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$S = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \alpha c \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \alpha c \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right)$$

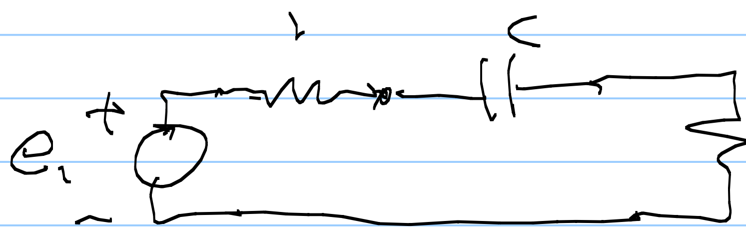
$$= \begin{bmatrix} \alpha c + 1 & -\alpha c \\ -\alpha c & \alpha c + 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 - \alpha c & \alpha c \\ \alpha c & 1 - \alpha c \end{bmatrix}$$

$$= \frac{1}{1 + 2\alpha c} \begin{bmatrix} 1 + \alpha c & \alpha c \\ \alpha c & 1 + \alpha c \end{bmatrix} \begin{bmatrix} 1 - \alpha c & \alpha c \\ \alpha c & 1 - \alpha c \end{bmatrix}$$

$$= \frac{1}{1 + 2\alpha c} \begin{bmatrix} 1 & 2\alpha c \\ 2\alpha c & 1 \end{bmatrix}$$

$$S^T(-\alpha)\, S(\alpha) = \frac{1}{1 - 2\alpha c} \begin{bmatrix} 1 & -2\alpha c \\ -2\alpha c & 1 \end{bmatrix} \frac{1}{1 + 2\alpha c} \begin{bmatrix} 1 & 2\alpha c \\ 2\alpha c & 1 \end{bmatrix}$$

$$= \frac{1}{1-4s^2c^2}\begin{bmatrix} 1-4s^2c^2 & 0 \\ 0 & 1-4s^2c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$v_2 = \frac{1}{2} S_{21} \cdot e_1$

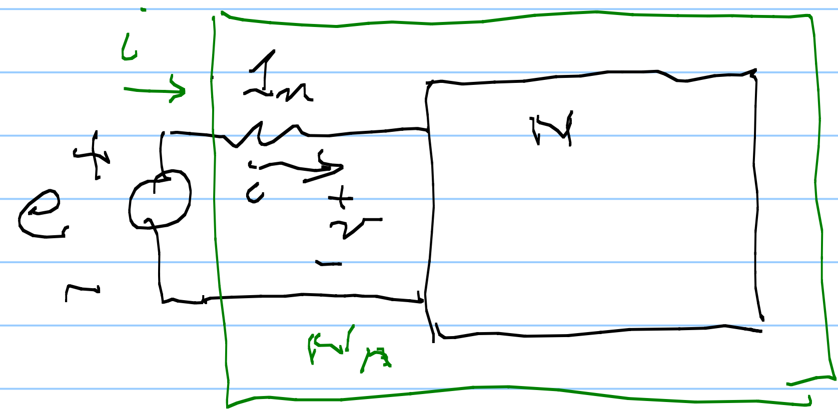
$$\frac{v_2}{e_1} = \frac{1}{2} \cdot \frac{2sc}{1+2sc} \quad \frac{1}{2+\frac{1}{sc}} = \frac{sc}{1+2sc} \quad \text{check}$$

Bounded real conditions $S(s)$

1. Real for real s in $\sigma > 0$
2. analytic in $\sigma > 0$ also $\sigma = 0$ if S is rational
3. $1_n - S^{T*}(\cancel{-}s)S(s)$ is positive semi-definite in $\sigma > 0$

The lossless condition is $1_m = S^T(-s)\,S(s)$

i.e. the inverse of $S(s)$ is $S^T(-s)$



$i = Y_A \cdot e$

$v = e - i = e - Y_A e$

$= (1_m - Y_A)e$

$$2v^i = v + i = Y_A e + (1_m - Y_A)e = e$$

$$2v^r = v - i = (1_m - Y_A)e - Y_A e = (1_m - 2Y_A)e$$

$$= (1_m - 2Y_A)\,2v^i \Rightarrow S = (1_m - 2Y_A)$$