EE 610.
10/14/14

Lossless: $Ev\, y(s) = 0 \Rightarrow Ev\, z(s) = 0 =$

$$2Ev(z(s)) = z(s) + z(-s) = \frac{1}{y(s)} + \frac{1}{y(-s)}$$

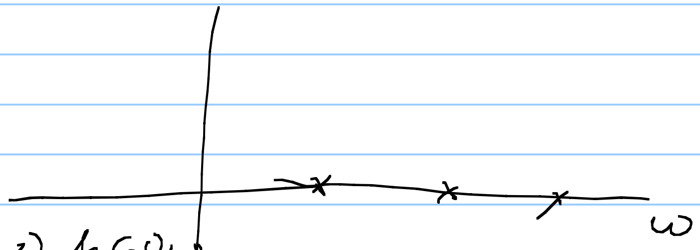
$$= \frac{y(-s) + y(s)}{y(s)\,y(-s)} = 0$$

Foster if PR ↓

$$y(s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{N} \frac{2k_i s}{s^2+\omega_i^2} \quad , \quad k_i \geq 0$$

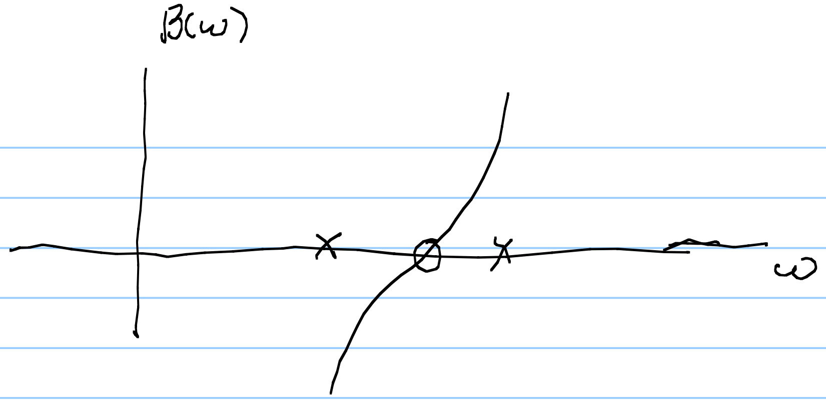
$$y(j\omega) = -j\frac{k_0}{\omega} + j\, k_\infty \omega + \sum_{i=1}^{N} \frac{2k_i j\omega}{-\omega^2+\omega_i^2}$$

$y(j\omega)/j = B(\omega)$



$$\frac{d\, y(j\omega)/j}{d\omega} = -\frac{k_0(-1)}{\omega^2} + k_\infty + 2\sum_{i=1}^{N} \frac{k_i}{(-\omega^2+\omega_i^2)} + \frac{(-2\omega)\, k_i(-1)\omega}{(-\omega^2+\omega_i^2)^2}$$

$$= \frac{k_0}{\omega^2} + k_\infty + 2\sum k_i\left[-\omega^2+\omega_i^2+2\omega^2\right]/(-\omega^2+\omega_i^2)^2 \geq 0$$

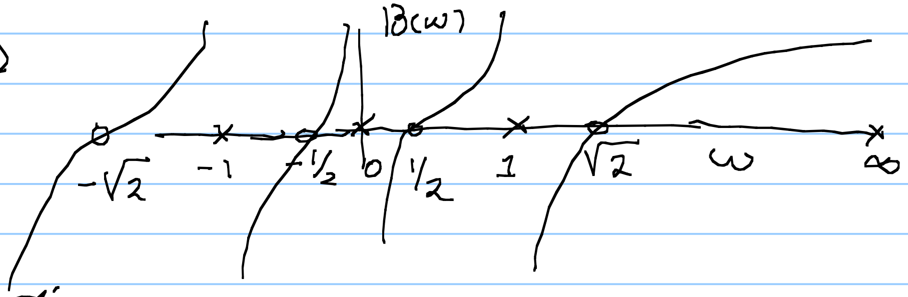


⇒ a zero between poles

∴ poles of $z(s)$ are zeros $y(s)$ between poles of $y(s)$

Ex: $y(s) = -y(-s)$ for lossless PR & poles & zeros alternate

$$= \frac{(s^2+1/4)(s^2+2)}{s(s^2+1)}$$



2nd Foster ⇒ partial fraction expansion of $y(s)$

$$y(s) = \frac{k_0}{s} + k_\infty s + \frac{2k_1 s}{s^2+1} = \frac{(s^2+1/4)(s^2+2)}{s(s^2+1)}$$

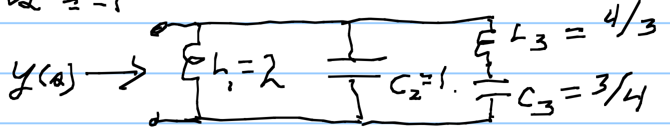
$k_0 \rightarrow \times s$ on both sides, cancel on right & set $s=0$ (kills k_∞ & k_1 terms

$$k_0 = \left.\frac{(s^2+1/4)(s^2+2)}{(s^2+1)}\right|_{s=0} = \frac{1/4 \times 2}{1} = 1/2$$

$$k_\infty = \frac{s^4}{s^3} \times \frac{1}{s} = 1$$

$$2k_1 = \left.\frac{\cancel{s^2+1}}{s}\left[\frac{(s^2+1/4)(s^2+2)}{s(\cancel{s^2+1})}\right]\right|_{s^2=-1} = \frac{(-1+1/4)(-1+2)}{-1} = \frac{-3/4 \times 1}{-1} = 3/4$$

$$y(s) = \frac{1/2}{s} + s + \frac{3/4\, s}{s^2+1}$$



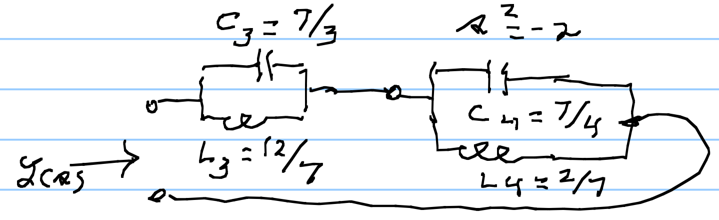
$$y_3 = \frac{3/4\, s}{s^2+1} \Rightarrow z_3 = \frac{1}{y_3} = \frac{s^2+1}{\frac{3}{4}s} = \frac{4}{3}s + \frac{1}{\frac{3}{4}s}$$

1st Foster ⇒ partial fraction expansion of $Z(s)$

$$Z(s) = \frac{1}{Y(s)} = \frac{1}{\frac{(s^2+1/4)(s^2+2)}{s(s^2+1)}} = \frac{s(s^2+1)}{(s^2+\frac{1}{4})(s^2+2)} = \frac{2k_3 s}{s^2+1/4} + \frac{2k_4 s}{s^2+2}$$

$$2k_3 = \frac{s^2+1}{s^2+2}\Bigg|_{s^2=-1/4} = \frac{3/4}{7/4} = 3/7 \quad , \quad 2k_4 = \frac{s^2+1}{s^2+1/4}\Bigg|_{s^2=-2} = \frac{-1}{-7/4} = 4/7$$

$$Z(s) = \frac{3s/7}{s^2+1/4} + \frac{4/7\, s}{s^2+2}$$



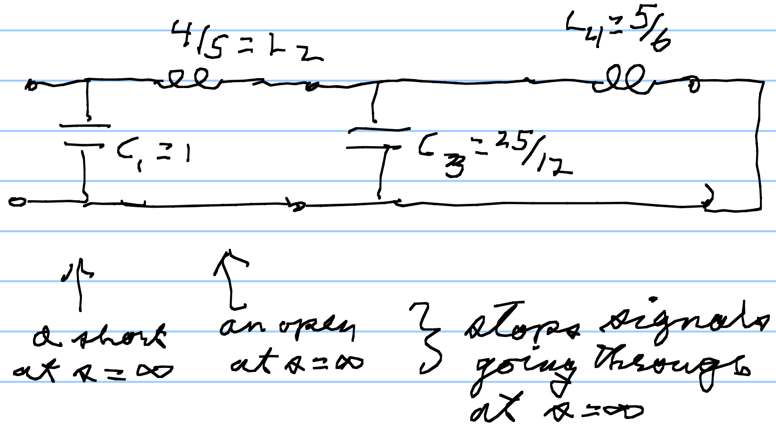
$$y_3 = \frac{s^2+1/4}{\frac{3}{7}s} = \frac{7}{3}s + \frac{7}{12s}, \quad y_4 = \frac{s^2+2}{\frac{4s}{7}} = \frac{7s}{4} + \frac{1}{\frac{2}{7}s}$$

1st Cauer: continued fraction expansion about ∞
(that is, remove poles at ∞ of y or z)

y(s) has a pole @ ∞ ; divide highest power of s

$$y(s) = \frac{(s^2+1/4)(s^2+2)}{s(s^2+1)} = \frac{s^4+\frac{9}{4}s^2+1/2}{s^3+s} \Rightarrow$$

$$= \underset{y\nearrow}{s} + \cfrac{1}{\underset{z\nearrow}{\tfrac{4}{5}s} + \cfrac{1}{\underset{y\nearrow}{\tfrac{25}{12}s} + \cfrac{1}{\underset{z\nearrow}{\tfrac{5}{6}s}}}}$$



$$s^3 + s \,\Big)\, s^4 + \frac{9}{4}s^2 + 1/2 \quad s$$

$$s^4 + s^2$$

$$\boxed{5/4\, s^2 + 1/2} \,\Big)\, s^3 + s \quad \frac{4}{5}s$$

$$s^3 + \frac{2}{5}s$$

$$\frac{3}{5}s \,\Big)\, \frac{5}{4}s^2 + 1/2 \quad \frac{25}{12}s$$

$$\frac{5}{4}s^2$$

$$1/2 \Big/ \frac{3}{5}s$$

2 nd Cauer ⇒ continued fraction expansion about $s = 0$

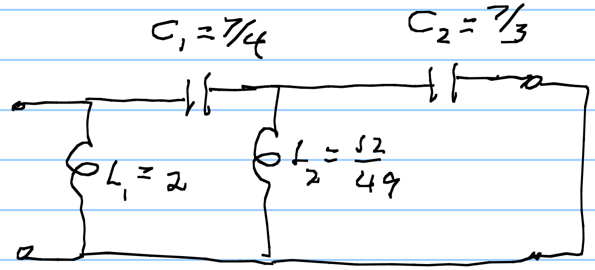
$$y(s) = \frac{\frac{1}{2} + \frac{9}{4}s^2 + s^4}{s + s^3} = \frac{1}{2s} + \cfrac{1}{\cfrac{1}{\frac{7}{4}s} + \cfrac{1}{\cfrac{1}{\frac{12}{49}\cdot\frac{1}{s} + \cfrac{1}{\cfrac{1}{\frac{7}{3}s}}}}}$$

$$\begin{array}{r|l} & \frac{1}{2s} \\ s+s^3 & \frac{1}{2} + \frac{9}{4}s^2 + s^4 \\ & \frac{1}{2} + \frac{1}{2}s^2 \\ \hline & \frac{7}{4}s^2 + s^4 \end{array}$$

$$\begin{array}{r|l} & \frac{1}{\frac{7}{4}s} \\ \frac{7}{4}s^2 + s^4 & s + s^3 \\ & s + \frac{4}{7}s^3 \\ \hline & \frac{3}{7}s^3 \end{array}$$

$$\begin{array}{r|l} & \frac{49}{12s} \\ \frac{3}{7}s^3 & \frac{7}{4}s^2 + s^4 \\ & \frac{7}{4}s^2 \\ \hline & s^4 \end{array}$$

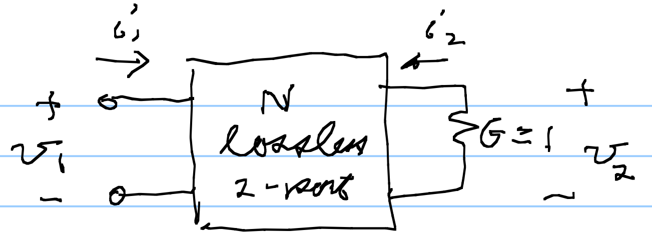
$$\begin{array}{r|l} & \frac{3}{7s} \\ s^4 & \frac{3}{7}s^3 \end{array}$$



open @ $s=0$ close @ $s=0$

useful for high pass

Use for v_2/v_1 synthesis



$$\begin{bmatrix} i_1 \\ -v_2 = i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$-v_2 = y_{21} v_1 + y_{22} v_2$$

$$\frac{v_2}{v_1} = \frac{-y_{21}}{1+y_{22}} \Rightarrow$$ given $y_{22}(s)$ is lossless, $y_{22} = \frac{N(s)}{D(s)}$ is odd

y_{22} is Even/odd or odd/even

$$= \frac{-k(\text{polynomial})}{\text{polynomial}} = \frac{-k(\text{odd or even})}{\text{Even} + \text{odd}} \Rightarrow \text{should be Hurwitz}$$

if numerator is odd divide out even's & (vice versa)

Ex: $\frac{V_2}{V_1}(s) = \frac{-k s^2}{s^4 + 2s^3 + 4s^2 + 2s + 2} = \frac{-k \frac{s^2}{2s^3+2s}}{1 + \frac{s^4+4s^2+2}{2s^3+2s}} = \frac{-y_{21}}{1 + y_{22}}$

$y_{22} = \frac{s^4 + 4s^2 + 2}{2s^3 + 2s}$ synthesize to get $y_{21} = -k \frac{s^2}{2s^3+2s}$