Note Title 9/28/2014

EE 610
09/30/14

$$\dot{x} = Ax + Bu \qquad x = T\hat{x} \quad ; \quad \dot{} = \frac{d}{dt} \qquad x = \text{state vector}$$

$$y = Cx + Du \qquad \text{assume } T^{-1} \text{ exists} \qquad \hat{x} = \text{new state}$$

$$\frac{dT}{dt} = 0_{k \times k}$$

$$T\dot{\hat{x}} = AT\hat{x} + Bu \qquad \Rightarrow \qquad \dot{\hat{x}} = T^{-1}AT\hat{x} + T^{-1}Bu$$

$$y = CT\hat{x} + Du \qquad\qquad y = CT\hat{x} + Du$$

$$T(s) = D + C(s1_k - A)^{-1}B = CT(s1_k - T^{-1}AT)^{-1}T^{-1}B$$

$$= CT\left(T^{-1}\left[Ts1_kT^{-1} - A\right]T\right)^{-1}T^{-1}B$$

$$= CT(T^{-1})(s1_k - A)^{-1}(T^{-1})^{-1} \cdot T^{-1}B$$

$$= C(s1_k - A)^{-1}B$$

Ex: $\ddot{x}_0 + 3\dot{x}_0 + 2x_0 = -2v_i$ $\quad$ x_0 = scalar, $u = v_i$

$y = 5x_0$ $\quad$ $y = i$ at the input

$x_1 = x_0$

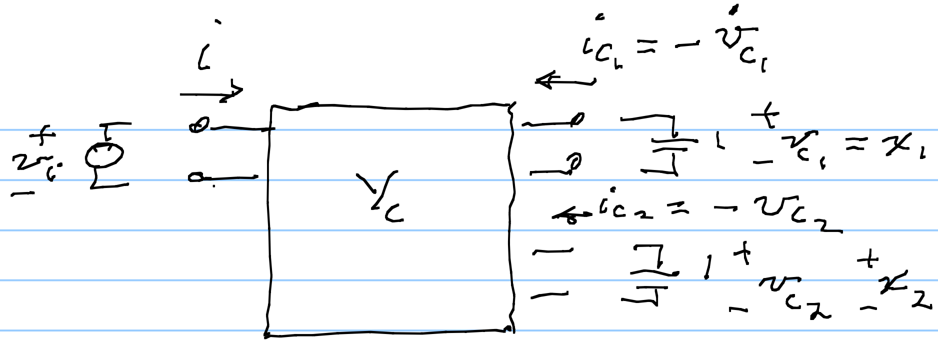
$x_2 = \dot{x}_1 = \dot{x}_0$ $\quad$ $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$x_3 = \dot{x}_2 = \ddot{x}_1 = \dddot{x}_0 = -3\dot{x}_0 - 2x_0 - 2v_i$

$= -3x_2 - 2x_1 - 2v_i$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \end{bmatrix} v_i, \quad i = y = \begin{bmatrix} 5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a circuit



$$\begin{bmatrix} i_{c_1} \\ i_{c_2} \\ i \end{bmatrix} = Y_c \begin{bmatrix} x_1 \\ x_2 \\ v_i \end{bmatrix} = \begin{bmatrix} -\dot{x}_1 \\ -\dot{x}_2 \\ i = y \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 2 & 3 & 2 \\ 5 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 = v_{c_1} \\ x_2 = v_{c_2} \\ u = v_i \end{bmatrix}$$

choose $T = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$; $\det T = t_{11} t_{22} - t_{12} t_{21} \neq 0$

$$T^{-1} = \frac{1}{\det T} \begin{bmatrix} t_{22} & -t_{12} \\ -t_{21} & t_{11} \end{bmatrix}$$

$$\hat{A} = T^{-1}AT, \quad \hat{B} = T^{-1}B, \quad \hat{C} = CT$$

$$\begin{bmatrix} -\dot{x} \\ i = y \end{bmatrix} = \begin{bmatrix} -A & -B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \Rightarrow \begin{bmatrix} -\dot{\hat{x}} \\ i = y \end{bmatrix} = \begin{bmatrix} -T^{-1}AT & -T^{-1}B \\ CT & D \end{bmatrix} \begin{bmatrix} \hat{x} \\ u \end{bmatrix}$$

$$Y_c = \begin{bmatrix} -A & -B \\ C & D \end{bmatrix} \simeq \hat{Y}_c = \begin{bmatrix} -T^{-1}AT & -T^{-1}B \\ CT & D \end{bmatrix}.$$

$$= \begin{bmatrix} T^{-1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -A & -B \\ C & D \end{bmatrix} \begin{bmatrix} T & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \Rightarrow T^{-1} \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} T = \frac{1}{\det} \begin{bmatrix} t_{22} & -t_{12} \\ -t_{21} & t_{11} \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$$

$$= \frac{1}{\det T} \begin{bmatrix} -2t_{12} & -t_{22} - 3t_{12} \\ 2t_{11} & t_{21} + 3t_{11} \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$$

$$= \frac{1}{\det T}\begin{bmatrix} -2t_{11}t_{12} - t_{22}t_{21} - 3t_{12}t_{21} & -2t_{12}^2 - t_{22}^2 - 3t_{12}t_{22} \\ 2t_{11}^2 + t_{21}^3 + 3t_{11}t_{21} & 2t_{11}t_{12} + t_{21}t_{22} + 3t_{11}t_{22} \end{bmatrix}$$

if desire $\hat{Y}_{c_{12}} = -\hat{Y}_{c_{21}} \Rightarrow 2t_{11}^2 + t_{21}^2 + 3t_{11}t_{21} = 2t_{12}^2 + t_{22}^2 + 3t_{12}t_{22}$ $\hat{(*)}$

lots of freedom but usually desire $\hat{Y}_{c_{ii}} \geq 0$ & $\hat{Y}_{c_{ij}} = -\hat{Y}_{c_{ji}}$, $i \neq j$

passive R's — use passive gyrators

Let $t_{21} = t_{12}$, $t_{11} = -1$; $(*) \Rightarrow 2 + t_{21}^2 - 3t_{21} = 2t_{21}^2 + t_{22}^2 + 3t_{21}t_{22}$

$$\Rightarrow t_{21}^2 + (3 + t_{22})t_{21} + (t_{22}^2 - 2) = 0$$

$$\Rightarrow t_{21} = \frac{-(3+t_{22})}{2} \pm \frac{1}{2}\sqrt{(9 + 6t_{22} + t_{22}^2) - 4t_{22}^2 + 8}$$

$$\Rightarrow t_{21} = \frac{-(3+t_{22})}{2} \pm \frac{1}{2}\sqrt{17+6t_{22}-3t_{22}^2}$$

if $t_{22} = 1 \Rightarrow t_{21} = -2 \pm \sqrt{\frac{20}{4}} = t_{12}, \quad t_{11} = -1$

$$T = \begin{bmatrix} -1 & -2+\sqrt{5} \\ -2+\sqrt{5} & 1 \end{bmatrix}, \quad T^{-1} = \frac{1}{1-(-2+\sqrt{5})^2} \begin{bmatrix} 1 & +2-\sqrt{5} \\ +2-\sqrt{5} & -1 \end{bmatrix}$$

$$\det = -1-(4-4\sqrt{5}+5) = -10+4\sqrt{5} = 2(-5+2\sqrt{5}) = 2\sqrt{5}(2-\sqrt{5})$$

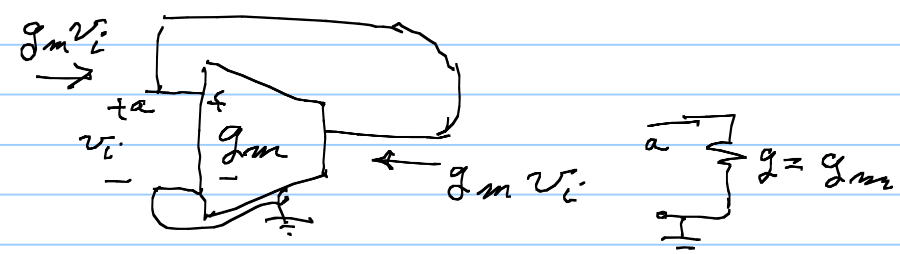
$$Y_c = Y_{c\,\text{symmetric}} + Y_{c\,\text{skew symmetric}} = Y_{cs} + Y_{csk}$$

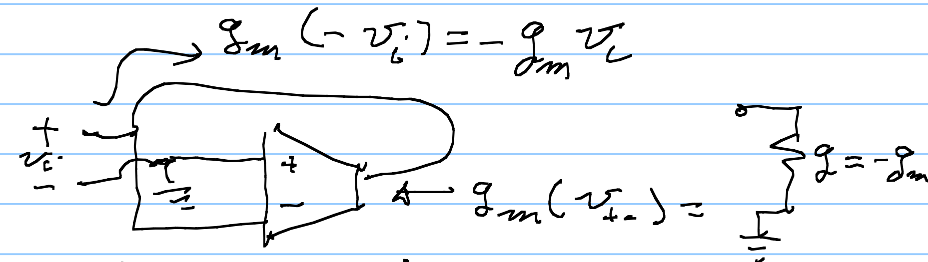
$$= \frac{Y_c + Y_c^T}{2} + \frac{Y_c - Y_c^T}{2}$$

$$Y_{cs}^T = Y_{cs}, \quad Y_{csk}^T = -Y_{csk}$$

$$Y_c = \begin{bmatrix} 0 & -1 & 0 \\ 2 & 3 & 2 \\ 5 & 0 & 0 \end{bmatrix} ; \quad 2Y_{c_a} = \begin{bmatrix} 0 & -1 & 0 \\ 2 & 3 & 2 \\ 5 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 5 \\ -1 & 3 & 0 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 5 \\ 1 & 6 & 2 \\ 5 & 2 & 0 \end{bmatrix} = 2Y_{c_a}^T$$

$$2Y_{c_{sk}} = \begin{bmatrix} 0 & -1 & 0 \\ 2 & 3 & 2 \\ 5 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 5 \\ -1 & 3 & 0 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -3 & -5 \\ 3 & 0 & 2 \\ 5 & -2 & 0 \end{bmatrix} = -2Y_{c_{sk}}^T$$

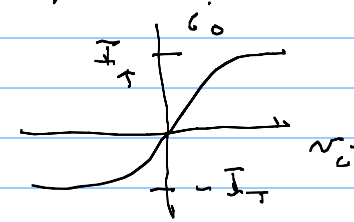




(a negative resistor)

If the OTA is operated large signal & is BJT

$$i_o = I_T \tanh(v_i/2V_T)$$



$$\tanh x = \frac{\frac{e^x - e^{-x}}{2}}{\frac{e^x + e^{-x}}{2}} = \frac{\sinh x}{\cosh x}$$

$$\frac{d \tanh x}{dx} = \frac{e^x + e^{-x}}{e^x + e^{-x}} - \frac{(e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2} = 1 - \tanh^2 x$$

Taylor series about $v_i = 0$

$$i_o = I_T \tanh\left(\frac{v_i}{2V_T}\right)\Big|_{v_i=0} + \frac{I}{2}\left(1 - \tanh^2\left(\frac{v_i}{2V_T}\right)\right) \cdot \frac{1}{2V_T} \cdot (v_i - 0) + \cdots$$

$$i_o = 0 + I_T(1-0)\cdot\frac{1}{2V_T}\cdot v_i + \cdots = \frac{I_T}{2V_T}\cdot v_i + \cdots$$

$$\approx g_m v_i \; ; \; g_m = \frac{I_T}{2V_T} \text{ mhos} = \mho \; , \quad \Omega = \text{ohms}$$