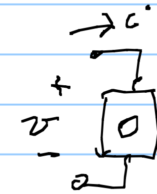
EE 610
09/16/14

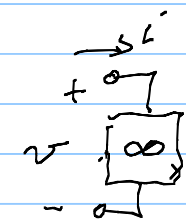
Note Title — 9/15/2014

strange devices: nullator

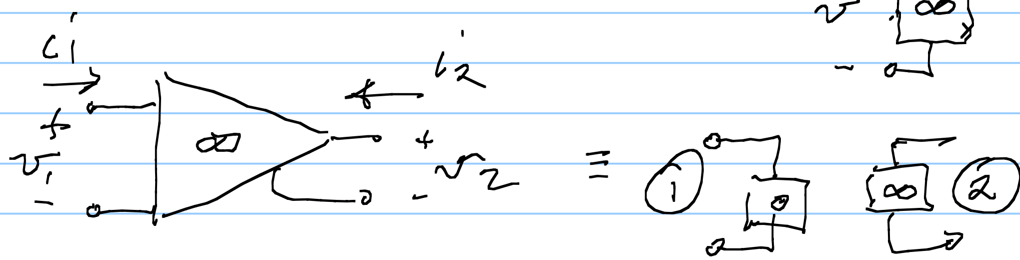


$v = 0$
$i = 0$

norator



i & v can be anything



semi-state equations

$$E\dot{x} = Ax + Bu$$
$$y = Cx$$

u = input
y = output
x = semi-state

How to get?

$$A_{bd} v_{bd} = B_{bd} i_{bd}$$ device equations (linear)

$$v_b = \mathcal{C}^T v_t \,, \quad i_b = \mathcal{J}^T i_\ell$$ $$v_b = v_{bd} + e \,, \quad i_b = i_{bd} + j$$

independent sources

$$\Rightarrow A_{bd} \cdot (v_b - e) = B_{bd} \cdot (i_b - j)$$

$$A_{bd} \cdot v_b - B_{bd} \cdot i_b = A_{bd} e - B_{bd} j$$

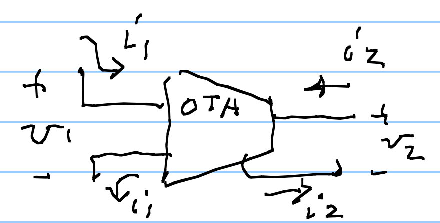
$$\underbrace{A_{bd} \cdot \mathcal{C}^T}_{b \times t} v_t - \underbrace{B_{bd} \cdot \mathcal{J}^T}_{b \times \ell} i_\ell = A_{bd} e - B_{bd} j$$ $\Leftarrow$ sources

$$\left[A_{bd} \cdot \mathcal{C}^T \;\vdots\; - B_{bd} \times \mathcal{J}^T \right] \begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = A_{bd} e - B_{bd} j = B \cdot u$$

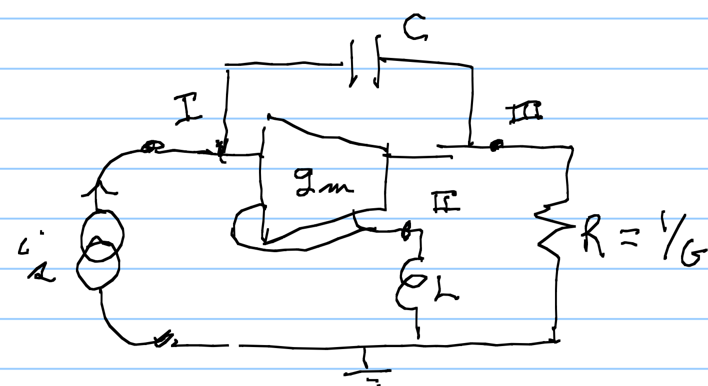
Let $x = \begin{bmatrix} v_t \\ \vdots \\ i_\ell \end{bmatrix}$; $\left[A_{bd}\, \mathcal{C}^T \;\vdots\; -B_{bd}\, \mathcal{J}^T \right] = \alpha \dot{\xi} + [-A]$

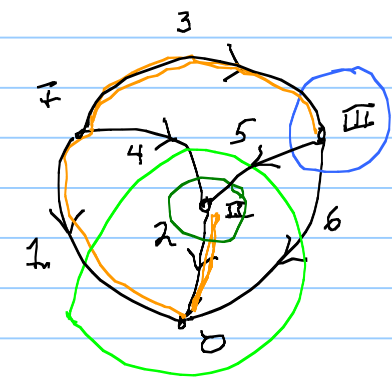
$=$

Ex:



$$= \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$





$$A_{bd} v_{bd} = B_{bd} i_{bd} \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & sC & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}_{bd} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & sL & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}_{bd}$$

$$i_b = i_{bd} + j$$

$$j = \begin{bmatrix} -i_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad e = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} i_s$$

$$\text{KCL} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{\begin{matrix} 1 & 1 & 1 \\ -1 & -1 & 0 \\ 0 & -1 & -1 \end{matrix}}_{K} \\ 0 & 0 & 1 & \end{array}\right] i_b \; ; \quad \text{KVL}: \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \left[\begin{array}{c|ccc} \underbrace{\begin{matrix} -1 & 1 & 0 \\ -1 & 1 & 1 \\ -1 & 0 & 1 \end{matrix}}_{-K^T} & 1 & 0 & 0 \\ & 0 & 1 & 0 \\ & 0 & 0 & 1 \end{array}\right] v_b$$

$$\left[\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \;\middle|\; -\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha L & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]\begin{bmatrix} v_t \\ i_l \end{bmatrix} = \begin{bmatrix} -1(\kappa-1) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} i_l'$$

$$= \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -\alpha L & -\alpha L & 0 \\ 0 & 0 & \alpha C & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ g_m & -g_m & 0 & 0 & -1 & 0 \\ G & 0 & -G & 0 & 0 & -1 \end{array}\right] x = B i_l'$$

let output be $v_1 = y$

$$s\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -L & -L & 0 \\ 0 & 0 & C & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -g_m & +g_m & 0 & 0 & 1 & 0 \\ -G & 0 & G & 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} i_s$$

$$y = v_1 = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]\, x$$

To solve note 4th row $\Rightarrow x_4 = 0$ so can delete 4th row & column.

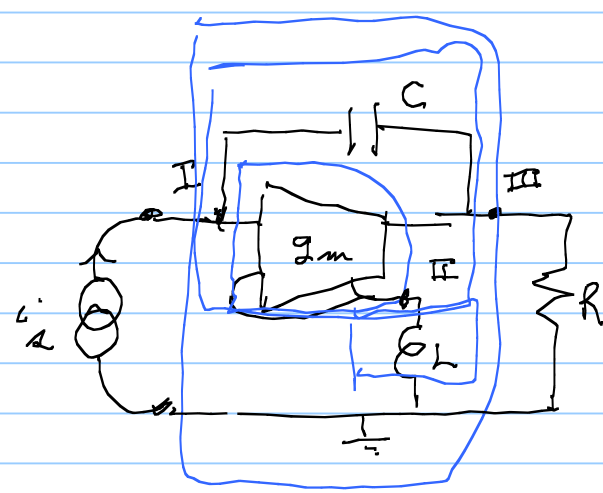
$$sEx = Ax + Bu, \quad y = Cx$$

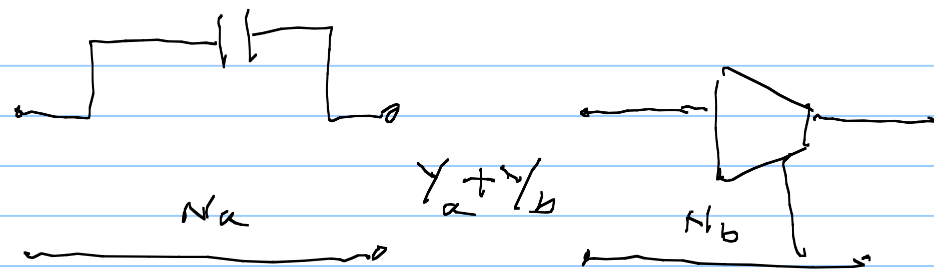
$$\Rightarrow (sE - A)x = Bu \Rightarrow x = (sE - A)^{-1} Bu$$

$$y = C[sE - A]^{-1} B \cdot u \Rightarrow v_1 = Z_{in} \cdot i_{in}\ ; \ Z_{in}(s) = C(sE - A)^{-1} B$$

$$\Rightarrow \; y_{in} = \frac{1}{z_{in}} = \frac{1}{C(sE-A)^{-1}B}$$

another way to get y_{in}:





Na
Ya + Yb
Nb