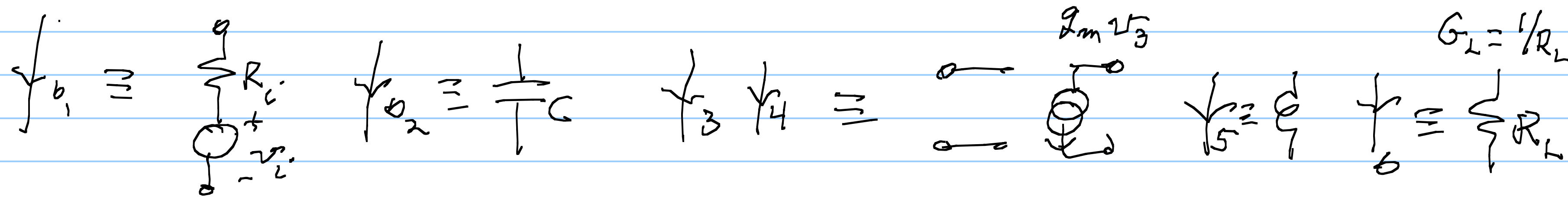
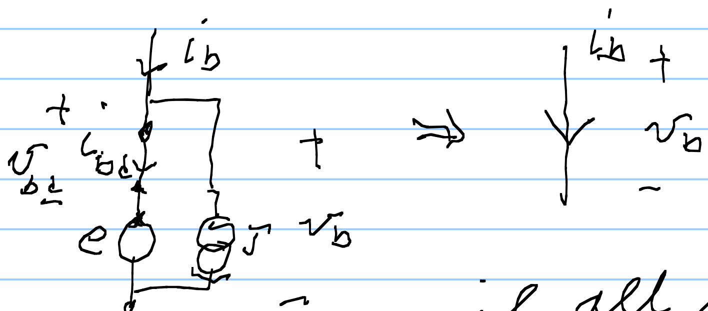
Note Title 9/6/2014

EE 610
09/11/14





$i_b = i_{bd} + J$

$v_b = v_{bd} + e$

J = independent current source

e = independent voltage sources

if all branches have admittances then

$$i_{bd} = Y_{b \times b} \cdot v_{bd}$$

$$i_{bd} = \begin{bmatrix} G_i & 0 & 0 & 0 & 0 & 0 \\ 0 & C\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/\alpha h & 0 \\ 0 & 0 & 0 & 0 & 0 & G_L \end{bmatrix} \begin{bmatrix} v_{bd_1} \\ v_{b_2} \\ v_{b_3} \\ v_{b_4} \\ v_{b_5} \\ v_{b_6} \end{bmatrix} ; \quad J = \underline{0}_6, \quad e = \begin{bmatrix} v_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \alpha = d^{[\cdot]}/dt$$

$$\left.\begin{array}{ll} i_{bd} = i_b - J & i_{bd} = Y_{b \times b} \cdot v_{bd} \\ v_{bd} = v_b - e & \end{array}\right\} \text{laws of components}$$

Laws for connection, KVL, KCL: $v_b = C^T v_t$, $i_b = \mathcal{J}^T i_\ell$

$$i_b - J = Y_{b \times b}(v_b - e) \Rightarrow \mathcal{J}^T i_\ell - J = Y_{b \times b}(C^T v_t - e) \quad (*)$$

also $P_{in}(t) = v_b^T i_b = (v_t^T (\mathcal{C}^T)^T) \mathcal{J}^T i_\ell = v_t^T \mathcal{C} \mathcal{J}^T i_\ell = 0$

for any circuit with the same graph choose independent voltage sources in the tree and independent current sources in the links $\Rightarrow$ v_t can be anything as can be i_ℓ $\Rightarrow \mathcal{C}\mathcal{J}^T = 0_{t \times \ell}$

multiply (4) by $\mathcal{C}$ = cut set matrix

$$\underbrace{\mathcal{C}\mathcal{J}^T}_{0} i_\ell - \mathcal{C}J = \mathcal{C} Y_{b\times b} \mathcal{C}^T v_t - \mathcal{C} Y_{b\times b} e$$

Norton's eq. current $\Rightarrow \underbrace{\mathcal{C} Y_{b\times b} e - \mathcal{C} J}_{i_N} = \mathcal{C} Y_{b\times b} \mathcal{C}^T v_t$ if $\mathcal{C} Y_{b\times b} \mathcal{C}^T$ is nonsingular we can solve for v_t

$\Rightarrow v_t = (\mathcal{C} Y_{b\times b} \mathcal{C}^T)^{-1} i_N \Rightarrow v_b = \mathcal{C}^T v_t$

$\Rightarrow v_{bd} = v_b - e$

$\Rightarrow i_{bd} = Y_{b\times b} v_{bd} \Rightarrow i_b = i_{bd} - J \Rightarrow$ solves the circuit

$\mathcal{C}\, Y_{6\times 6}\, \mathcal{C}^T$ is a $t \times t$ matrix

$$\mathcal{C} \underset{6\times 6}{Y} \mathcal{C}^T = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} G_i & 0 & 0 & 0 & 0 & 0 \\ 0 & Cs & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/sL & 0 \\ 0 & 0 & 0 & 0 & 0 & G_L \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} G_i & 0 & g_m & 0 & 0 & G_L \\ 0 & Cs & -g_m & 0 & 0 & -G_L \\ 0 & 0 & -g_m & 0 & 1/sL & -G_L \end{bmatrix}}_{\mathcal{C} Y_{6\times 6}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} G_i + g_m + G_L & -g_m - G_L & -G_L \\ -G_L - g_m & Cs + g_m + G_L & G_L \\ -g_m - G_L & g_m + G_L & \frac{1}{sL} + G_L \end{bmatrix}$$

gives i_t vs v_t $\Rightarrow$ $i_t \to i_N' = C\,Y_{6\times 6}\cdot e = \begin{bmatrix} G_i & 0 & g_m & 0 & 0 & G_L \\ 0 & C\alpha & -g_m & 0 & 0 & -G_L \\ 0 & 0 & -g_m & 0 & 1/\alpha_2 & -G_L \end{bmatrix} \begin{bmatrix} v_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} G_i \cdot v_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} G_i \cdot v_i \\ \hline 0 \\ 0 \end{bmatrix} = \begin{bmatrix} G_1 v_i \\ \hline 0_2 \end{bmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right] \begin{bmatrix} v_{b1} \\ \hline v_{b2} \\ v_{b5} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = Y_{21} v_{b1} + Y_{22} \begin{bmatrix} v_{b2} \\ v_{b5} \end{bmatrix} \Rightarrow -Y_{22}^{-1} Y_{21} v_{b1} = \begin{bmatrix} v_{b2} \\ v_{b5} \end{bmatrix}$$

$$\Rightarrow \quad G_1 v_1 = Y_{11} v_{b1} + Y_{12}\left(-Y_{22}^{-1} Y_{21}\right) v_{b1} = \Big[\underbrace{Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}}_{1\times 1}\Big] v_{b1}$$

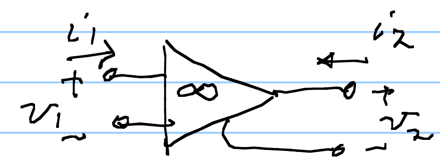
$\therefore$ need $Y_{22}^{-1} = \begin{bmatrix} Cs+g_m+G_L & G_L \\ g_m+G_L & \frac{1}{sL}+G_L \end{bmatrix}^{-1} =$

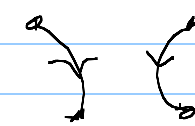
$$\frac{1}{(Cs+g_m+G_L)(\frac{1}{sL}+G_L)-G_L(g_m+G_L)} \begin{bmatrix} \frac{1}{sL}+G_L & -G_L \\ -(g_m+G_L) & Cs+g_m+G_L \end{bmatrix}$$

$$Y_{11}-Y_{12}Y_{22}^{-1}Y_{21} = [G_L+g_m+G_L] - \frac{[-g_m-G_L,\ -G_L] \begin{bmatrix} \frac{1}{sL}+G_L & -G_L \\ -(g_m+G_L) & Cs+g_m+G_L \end{bmatrix} \begin{bmatrix} G_L-g_m \\ -g_m-G_L \end{bmatrix}}{(Cs+g_m+G_L)(\frac{1}{sL}+G_L)-G_L(g_m+G_L)}$$

Example of no admittance device:

ideal op-amp





$\left.\begin{array}{l} i_1 = 0 \\ v_1 = 0 \end{array}\right\}$ law can't write $i = Y v$ $\Rightarrow$ so need more general

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow A v_{bd} = B i_{bd}$$