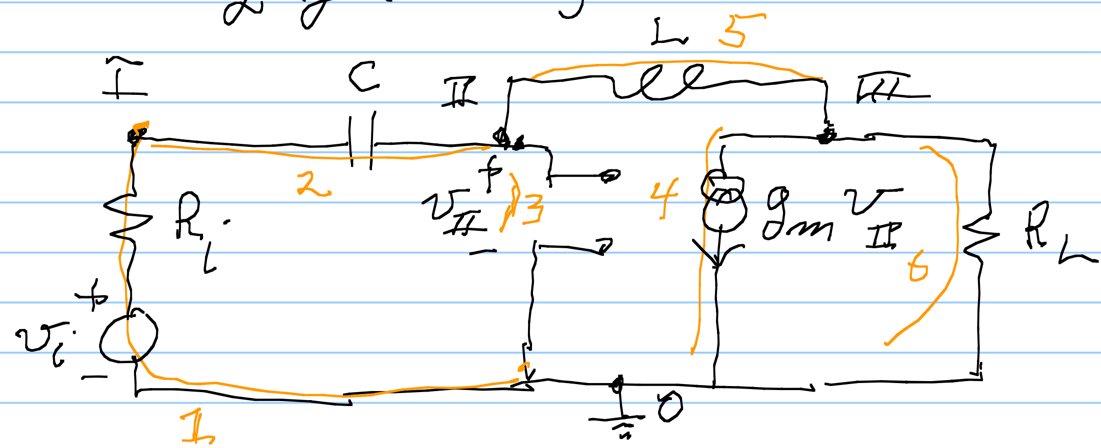
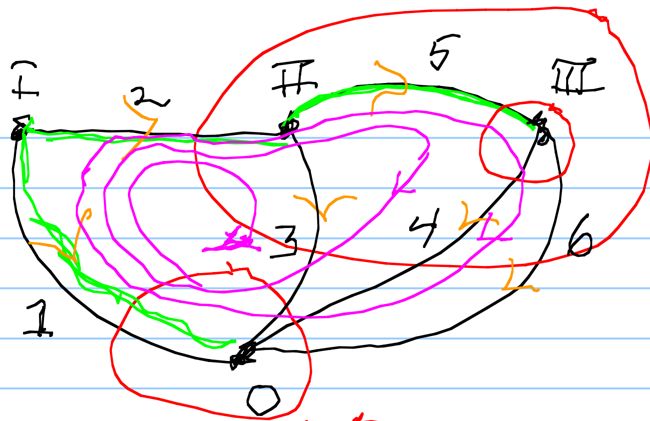
Note Title 9/6/2014

EE610
09/09/14

Graph Theory for KCL, KVL





2nd cut set $= b_2, b_3, b_4, b_6$

1st cut set $= b_1, b_3, b_4, b_6$

$n = \#$ of nodes $= 4$

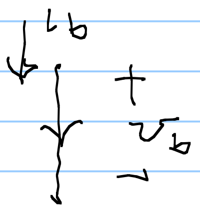
$b = \#$ of branches $= 6$

$t = \#$ of tree branches $= 3$

$l = \#$ of links $= 3$

$t = n-1$ $\qquad s = \#$ seperate parts

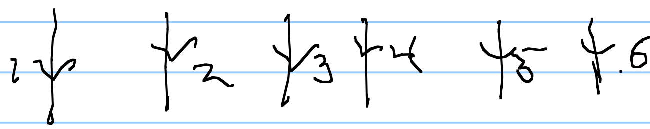
$l = b - t$ $\qquad = 1 \quad (t = n - s)$



; Power into branch $= v_b i_b \Rightarrow v_b^T i_b =$ total power into the circuit

$V^T =$ transpose

$\Rightarrow$



$i_b = [i_{b_1}, i_{b_2}, i_{b_3}, i_{b_4}, i_{b_5}, i_{b_6}]$

KCL: Σ currents into a closed sphere $= 0$

do for each tree branch $\Rightarrow$ 3 KCL's

$$0 = i_{b_1} + i_{b_3} + i_{b_4} + i_{b_6}$$

$$0 = i_{b_2} - i_{b_3} - i_{b_4} - i_{b_6}$$

$$0 = -i_4 + i_{b_5} - i_{b_6}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_{b_1} \\ i_{b_2} \\ i_{b_3} \\ i_{b_4} \\ i_{b_5} \\ i_{b_6} \end{bmatrix}$$

$\Rightarrow \quad \underline{0} = C \cdot i_b$; C = cut set matrix

$\quad$ & independent as 1 per tree branch

KVL: Σ voltages around a closed path $= 0$

do one for each link

b_3 $\quad 0 = -v_{b_1} + v_{b_2} + v_{b_3}$

b_4 $\quad 0 = -v_{b_1} + v_{b_2} + v_{b_4} + v_{b_5}$

b_6 $\quad 0 = -v_{b_1} + v_{b_2} + v_{b_5} + v_{b_6}$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 1 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}}_{\underline{0} = \mathcal{T} v_b} v_b$$

have l independent equations

$\mathcal{T}$ = tie set matrix

If number trees first can get equations

$$\underline{0}_{\tau} = \left[1_t \;\vdots\; K_i \right] \begin{bmatrix} i_t \\ i_l \end{bmatrix} \Rightarrow \quad 1_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ for } t = 3$$

$$\Rightarrow \quad -i_t = K_i i_l$$

& for links $\quad \underline{0}_{l} = \left[K_v \;\vdots\; 1_l \right] \begin{bmatrix} v_t \\ v_l \end{bmatrix} \Rightarrow -v_l = K_v v_t$

$$\begin{bmatrix} v_t \\ v_\ell \end{bmatrix} = v_b = \begin{bmatrix} 1_t \\ -K_v \end{bmatrix} \begin{bmatrix} v_t \end{bmatrix}; \quad \begin{bmatrix} i_t \\ i_\ell \end{bmatrix} = \begin{bmatrix} -K_i \\ 1_\ell \end{bmatrix} \begin{bmatrix} i_\ell \end{bmatrix}$$

as $P_{in} \equiv 0$ if a finite graph for a circuit then

$$v_b^T \cdot i_b = \sum_{k=1}^{b} v_{b_k} i_{b_k} = 0 \Rightarrow \underbrace{v_t^T \left[1_t \,\vdots\, -K_v^T \right]}_{v_b^T} \begin{bmatrix} -K_i \\ 1_\ell \end{bmatrix} i_\ell$$

$$\Rightarrow v_t^T \underbrace{\left[-K_i - K_v^T \right]}_{t \times \ell} i_\ell = 0_{1\times 1}$$

as v_t & i_ℓ can be for any circuit with this graph then this means $-K_i - K_v^T = 0_{t \times \ell}$

$\Rightarrow K_i = -K_v^T$ $\qquad \Rightarrow \mathcal{C} = [1_t, K_i]$; $\mathcal{C}^T = \begin{bmatrix} 1_t \\ -K_v \end{bmatrix} = \begin{bmatrix} 1_t \\ K_i^T \end{bmatrix}$

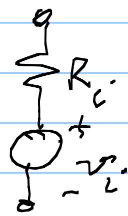
$\mathcal{J} = [K_v, 1_\ell]$; $\mathcal{J}^T = \begin{bmatrix} K_v^T \\ 1_\ell \end{bmatrix} = \begin{bmatrix} -K_i \\ 1_\ell \end{bmatrix}$

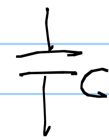
$$v_b = \begin{bmatrix} 1_t \\ -K_v \end{bmatrix} v_t = \begin{bmatrix} 1_t \\ K_i^T \end{bmatrix} v_t$$

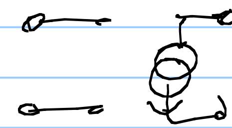
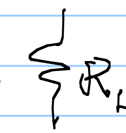
$\Rightarrow v_b = \mathcal{C}^T v_t$ $\qquad \Rightarrow i_b = \mathcal{J}^T i_\ell$

(KVL) $\qquad$ (KCL)

need laws of branches

$Y_{b_1} \equiv$ 

$Y_{b_2} \equiv$ 

$Y_{b_3}\, Y_{b_4} \equiv$ 
$Y_{b_5} \equiv$ 
$Y_{b_6} \equiv$ 

use admittances $\Rightarrow$ $Y_{6\times 6}$ as have an open circuit branch on VCCS