ENEE610: Homework 6 solutions

Problem 1

a) From homework 5:

$$T_n(s) = \frac{1}{(s-e^{5\pi/8})(s-e^{-5\pi/8})(s-e^{25\pi/8})(s-e^{-25\pi/8})} \xrightarrow{\text{expand}} T_n(s) = \frac{1}{(s^2+0.7654s+1)(s^2+1.8478s+1)} = T_{A0}(s)\times T_{B0}(s)$$

Compare to general 2nd order transfer function:

$$T_n(s) = \frac{k_0\omega_0}{\left(s^2+\frac{\omega_0}{Q}s+\omega_0^2\right)}$$

T_{A0}(s):

$\omega_0 = 1$

$$\frac{\omega_0}{Q} = 0.7654 \longrightarrow Q = \frac{1}{0.7654} = 1.3065$$

T_{B0}(s):

$\omega_0 = 1$

$$\frac{\omega_0}{Q} = 1.8478 \longrightarrow Q = \frac{1}{1.8478} = 0.5412$$

b) Denormalize T_n(s) to $\omega_C = 2\pi 10^4$ rads^{-1}:

$$T_{dn}(s) = \frac{1}{\left(\left(\frac{s}{\omega_C}\right)^2+0.7654\left(\frac{s}{\omega_C}\right)+1\right)\left(\left(\frac{s}{\omega_C}\right)^2+1.8478\left(\frac{s}{\omega_C}\right)+1\right)}$$

$$T_{dn}(s) = \frac{\omega_C^2}{s^2+0.7654s\omega_C+\omega_C^2}\times\frac{\omega_C^2}{s^2+1.8478s\omega_C+\omega_C^2} = T_A(s)\times T_B(s)$$

Recall 2nd order transfer function for the UAF42 (from class)

$$T(s) = \frac{\left(-\frac{RG_i}{C^2R_1R_2}\right)}{\left(s^2+s\left(\frac{2+RG_i}{2R_1C}\right)+\frac{1}{C^2R_1R_2}\right)}$$

We need to find values for R_1, R_2, and $R_i = 1/G_i$ for each 2nd order transfer function:

There is no constraint on DC gain for this problem, so let's set DC gain to -1:

$$T(0) = -1 = \left(-\frac{RG_i}{C^2R_1R_2}\right)(C^2R_1R_2) \longrightarrow G_i = \frac{1}{R} \longrightarrow \boldsymbol{R_i = R = 50\ k\Omega}$$

Compare T(s) and T_A(s):

$$s\frac{2+RG_i}{2R_{1A}C} = 0.7654s\omega_C \longrightarrow \boldsymbol{R_{1A} = \frac{2+RG_i}{2C \times 0.7654\omega_C} = 31.19\ k\Omega}$$

$$\frac{1}{C^2R_{1A}R_{2A}} = \omega_C^2 \longrightarrow \boldsymbol{R_{2A} = \frac{1}{C^2\omega_C^2R_{1A}} = 8.121\ k\Omega}$$

Compare T(s) and T_B(s):

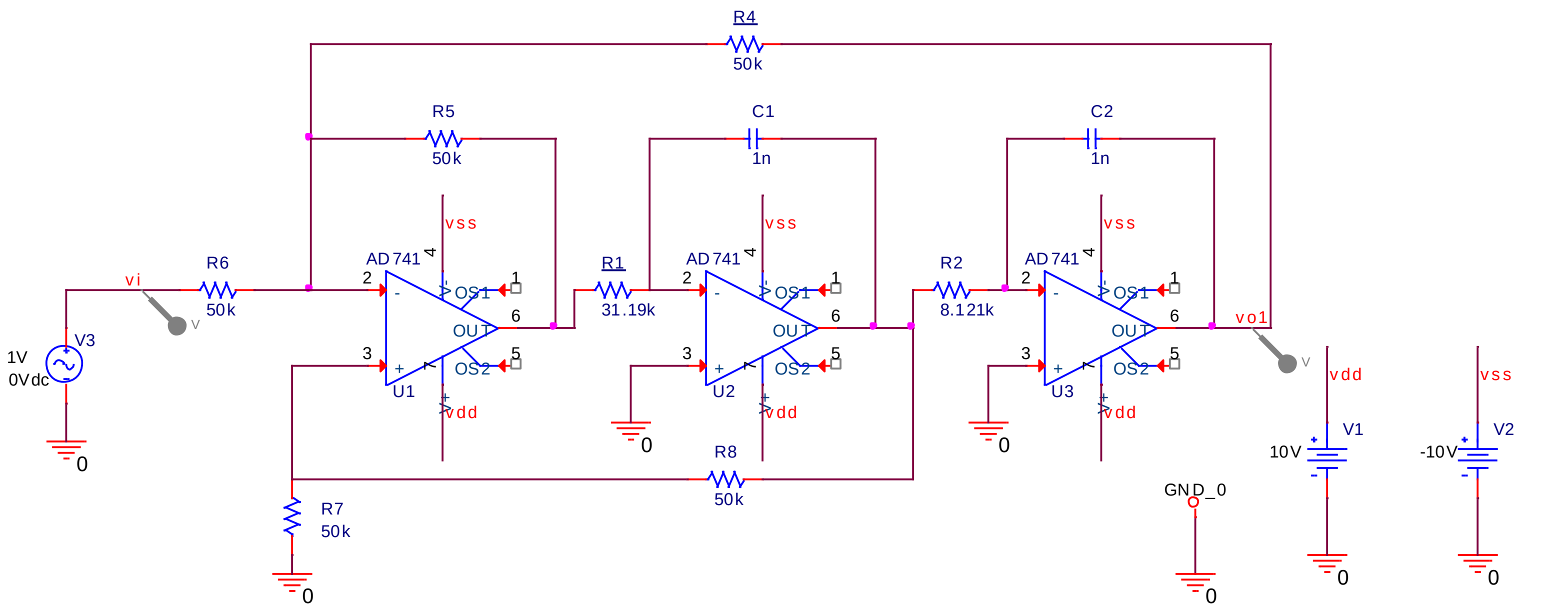
$$s\frac{2+RG_i}{2R_{1B}C} = 1.8478s\omega_C \longrightarrow \boldsymbol{R_{1B} = \frac{2+RG_i}{2C \times 1.8478\omega_C} = 12.919\ k\Omega}$$

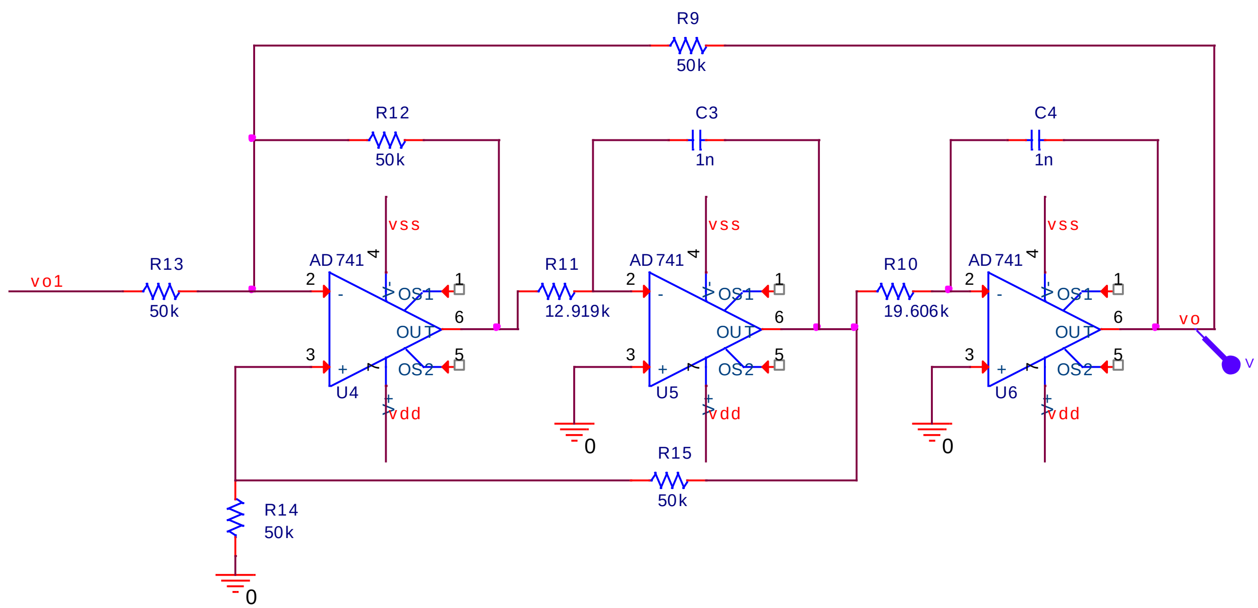
$$\frac{1}{C^2R_{1B}R_{2B}} = \omega_C^2 \longrightarrow \boldsymbol{R_{2B} = \frac{1}{C^2\omega_C^2R_{1B}} = 19.606\ k\Omega}$$

Summary of component values:

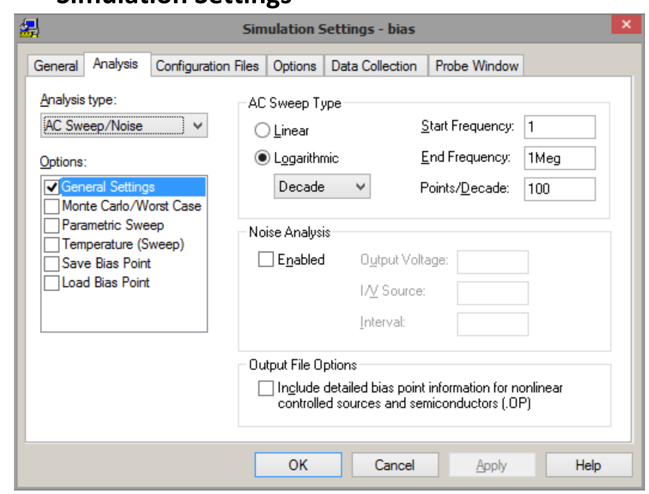
Component	First Stage T_A(s)	Second Stage T_B(s)
R_1	31.19 kΩ	12.919 kΩ
R_2	8.121 kΩ	19.606 kΩ
R_i	50 kΩ	50 kΩ

The resulting circuit schematic will be:

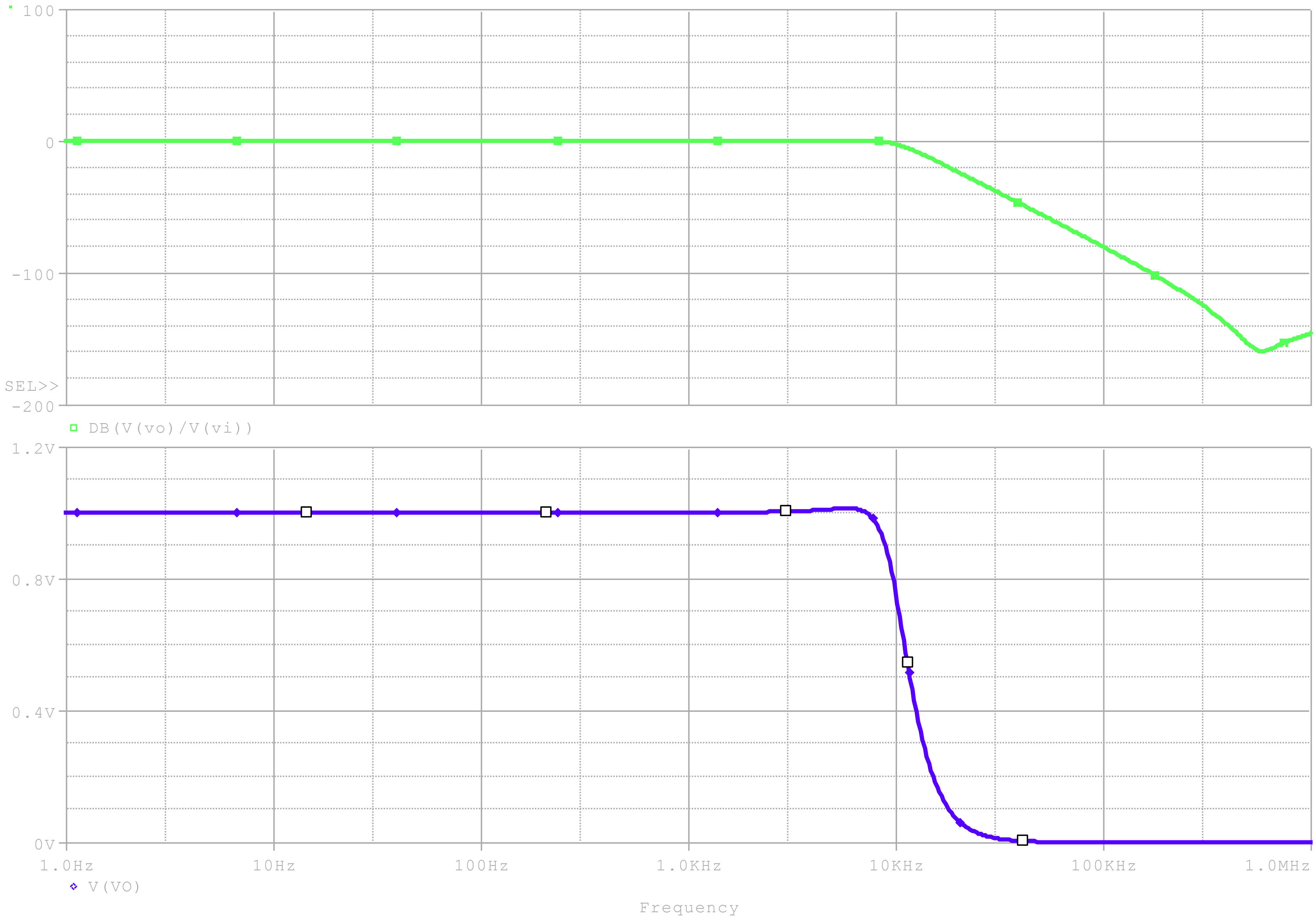




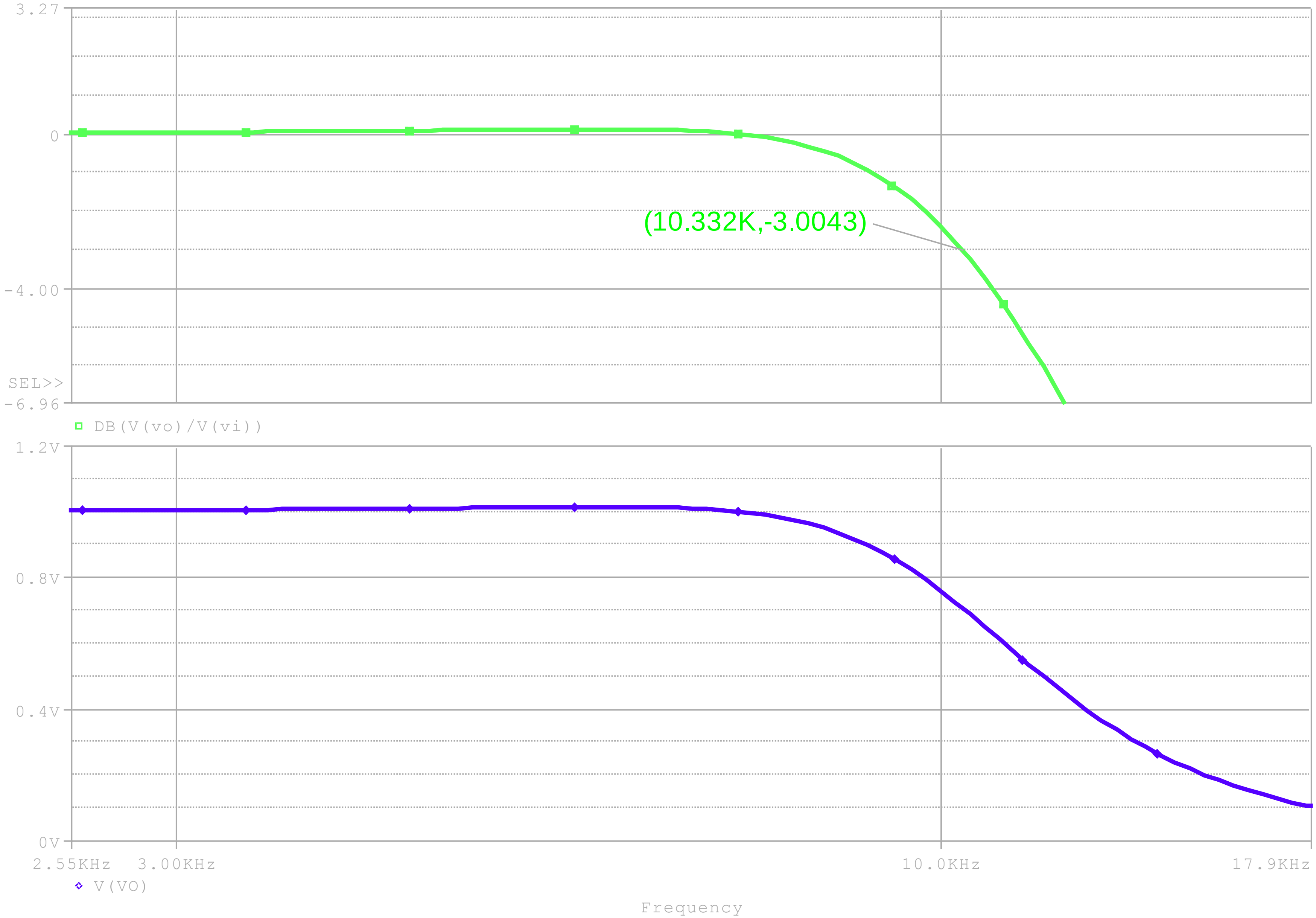
Simulation Settings



PSpice simulation:



A closer look around 10 kHz:



From the plot, our 3dB point is about 10.3 kHz.

Problem 2

$$y(s) = \frac{s^2 + as + 1}{s^2 + 2s + b}$$

a) Values of a, b such that y(s) is PR.

Since y(s) is a rational function we may use the following conditions to test for PR:

(a) no poles in the RHP

(b) poles on $j\omega$ axis are simple w/ real and positive residues

(c) $Re\{y(j\omega)\} \geq 0$ for $0 \leq \omega \leq \infty$

[Theorem 8.2-2, Peikari pg. 339]

Additional conditions for PR

(d) Coefficients of N(s) and D(s) are real and positive $\left[y(s) = \frac{N(s)}{D(s)}\right]$

(e) Zeros of N(s) and D(s) are in LHP

[Peikari, pg. 340]

Conditions (a), (d), (e):

Poles of y(s): $s^2 + 2s + b = 0 \Rightarrow s = -1 \pm \sqrt{1-b}$

Zeros of y(s): $s^2 + as + 1 = 0 \Rightarrow s = -\frac{a}{2} \pm \frac{\sqrt{a^2-4}}{2}$

$\left.\begin{matrix} a \geq 0 \\ b \geq 0 \end{matrix}\right\}$ for poles, zeros to be in LHP

Condition (c)

$$y(j\omega) = \frac{-\omega^2 + ja\omega + 1}{-\omega^2 + j2\omega + b}$$

$$Re\{y(j\omega)\} = \frac{(1-\omega^2)(b-\omega^2) + 2a\omega^2}{(b-\omega^2)^2 + (2\omega)^2} \geq 0$$

Denominator is positive $\therefore$ look at numerator:

$$f(\omega^2) = \omega^4 + \omega^2(2a-1-b) + b \geq 0$$

$\Rightarrow$ Quadratic in ω^2

$\rightarrow$ find minima:

$$\frac{df(\omega^2)}{d(\omega^2)} = 0 = 2\omega_0^2 + 2a - 1 - b \longrightarrow \omega_0^2 = \frac{b+1-2a}{2} \Rightarrow$$

If minima is < 0, $f(\omega^2) > 0$ for $\omega > 0$.
Since $\frac{df(\omega_i^2)}{\partial(\omega_i^2)} > 0$ for $\omega > \omega_0$ and $f(0) \geq 0$

If minima is > 0 then discriminant of $f(\omega^2)$ must be < 0

$\Rightarrow \Delta = (2a-1-b)^2 - 4b < 0$

Since we consider ω in $0 \leq \omega \leq \infty \Rightarrow \omega^2 \geq 0$

$\therefore$ the discriminant condition must hold

Summary

$a \geq 0 \qquad b \geq 0 \qquad (2a-1-b)^2 < 4b$

for $y(s)$ to be PR

b) For a, b such that $y(s)$ is PR:

Poles : $s = -1 \pm \sqrt{1-b}$

Zeros : $s = \frac{-a}{2} \pm \frac{\sqrt{a^2-4}}{2}$

$\Rightarrow$ pole can be on $j\omega$ axis at $s=0$ if $b=0$

$\Rightarrow$ zero can be on $j\omega$ axis at $s = \pm j$ if $a=0$