Note Title 11/6/2014

EE303H

11/06/14

Van der Pol

$$\frac{d^2x}{dt^2} + \mu(x^2-1)\frac{dx}{dt} + \omega_0^2 x = 0 \qquad x(0),\ \dot{x}(0)$$

$\mu \geq 0$

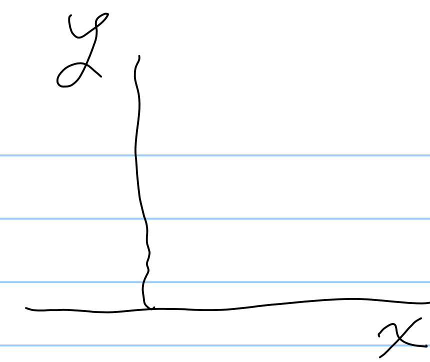
$$f(x) = \frac{\mu}{3}x^3 - \mu x$$

$$\frac{d f(x)}{dt} = \mu(x^2-1)\frac{dx}{dt}$$

$$\frac{d(y)}{dt} + \omega_0^2 x = 0, \qquad y = \frac{dx}{dt} + f(x)$$

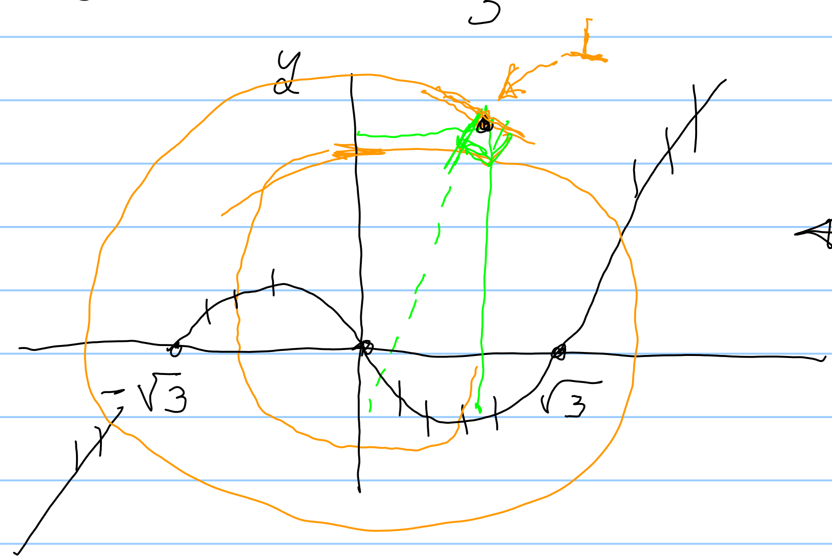
$$\left.\frac{dx}{dt} = y - f(x), \quad \frac{dy}{dt} = -\omega_0^2 x \right\} \text{state variable eqn.}$$

plot in y vs x for the solution



$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\omega_0^2 x}{y - f(x)}$$

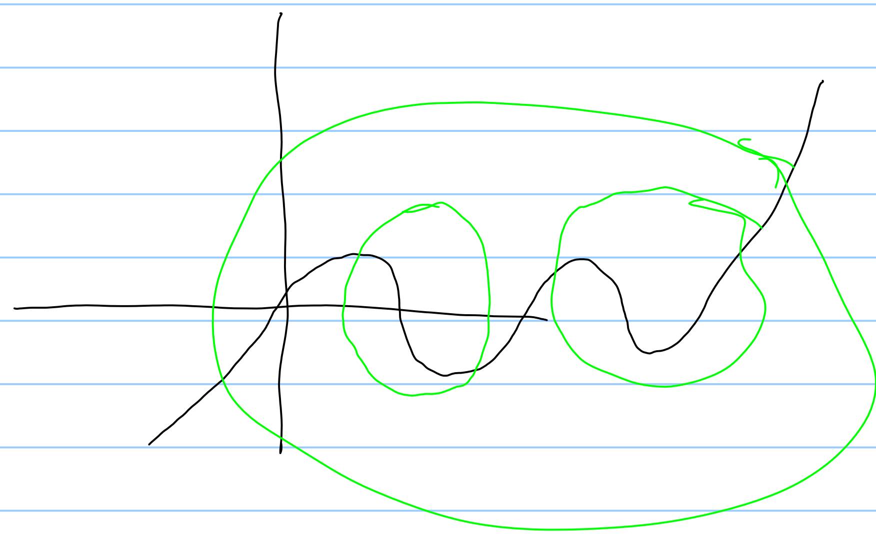
$$y = f(x) = \frac{\mu}{3}(x^3 - 3x) = \frac{\mu}{3} x (x^2 - 3)$$



goes to a limit cycle

x if normalize $\omega_0 = 1$

for multiple limit cycles use an $f(x)$
with two negative slopes



synthesis of passive circuits

$\mathcal{E}(t) \geq 0$ for all time

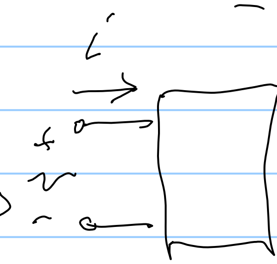
$\shortparallel$

energy into circuit $\mathcal{E}(t) = \int_{-\infty}^{t} p(\tau)\,d\tau$

$\shortparallel$

power

$$\mathcal{E}(t) = \int_{-\infty}^{t} v(\tau)\,i(\tau)\,d\tau$$



$$y(s) = \frac{V(s)}{I(s)}$$

$$V(s) = \int_{-\infty}^{\infty} v(t)\,e^{-st}\,dt = \text{Laplace transform}$$

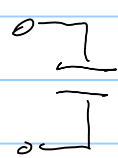
$y(s)$ will come from a passive circuit if it is positive real; can synthesize if $y(s)$ is rational & positive real $\Rightarrow$ PR

$f(s)$ by definition it is positive real if and only if

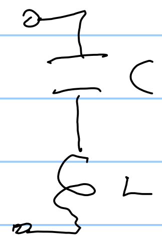
1. $f(s)$ is if s is real & $\mathrm{Re}\, s = \sigma > 0$
   (a real circuit)
2. $f(s)$ is analytic in $\sigma > 0$ (stable)
3. $\mathrm{Re}\, y(s) \geq 0$ in $\sigma > 0$

Ex:

R ; $y(s) = 1/R \Rightarrow$ PR if $R > 0$

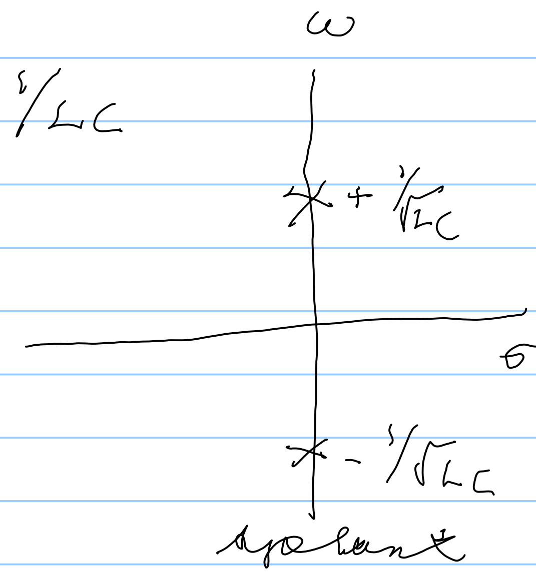


C $\quad y(s) = Cs$; $\mathrm{Re}\, y(s) = C(\sigma + \cancel{j\omega}) = C\sigma$ real if C is real
in $\sigma > 0$



$$y(s) = \frac{1}{Z(s)} = \frac{1}{sL + \frac{1}{sC}} = \frac{sC}{1 + LCs^2}$$

C & L real j poles at $s^2 = -1/LC$



$$s = \pm j/\sqrt{LC}$$

if L & $C > 0$ is stable

$\mathrm{Re}\, y(s)$ in $s = \sigma + j\omega$, $\sigma > 0$

$$y(s) = \frac{(\sigma + j\omega)C}{1 + LC(\sigma^2 + 2j\omega\sigma - \omega^2)}$$

$$= \frac{(\sigma + j\omega)C(1 + LC\sigma^2 - LC\omega^2 - 2j\omega LC\sigma)}{[1 + LC\sigma^2 - LC\omega^2 + 2j\omega LC\sigma][1 + LC\sigma^2 - LC\omega^2 - 2j\omega LC\sigma]}$$

$$\mathrm{Re}\, y(s) \times D(s) D(s^*) = \sigma\left[1 + LC\sigma^2 + 2\omega^2 LC - LC\omega^2\right] = \sigma(1 + LC\sigma^2)$$

$$> 0 \text{ in } \sigma > 0$$

$$\Rightarrow \quad y(s) = \frac{Cs}{LCs^2 + 1} \text{ is PR}$$

How to make a circuit

Lossless $\Rightarrow P_{ave}(j\omega) = 0$

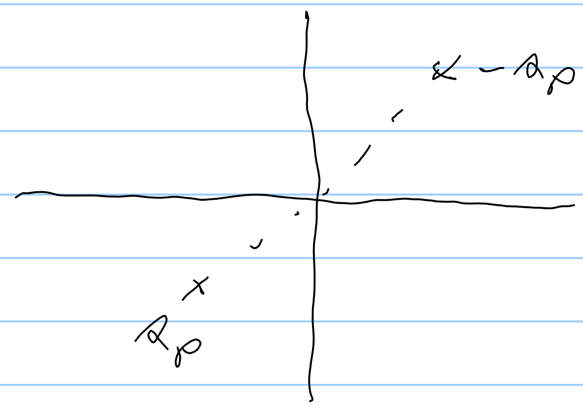
$$V^*(s) I(s)\Big|_{s=j\omega} + V(s) I^*(s)\Big|_{s=j\omega} = 2 P_{ave}(j\omega) \overset{\text{Lossless}}{=} 0$$

$$V^* y(s) V + V y^*(s) V^* = V^*\left[y(s) + \underset{s=j\omega}{y^*(s)}\right] V = 0$$

$$\Rightarrow \quad V^*\left[y(s) + y(-s)\right]\Big|_{s=j\omega} V = 0 \quad \text{for all } V$$

$$\Rightarrow \quad y(s) + y(-s) = 0 \quad \text{for } s = j\omega \Leftarrow \text{lossless}$$

holds for all $s = \sigma + j\omega \Rightarrow Y(s) = -Y(-s)$ for a lossless PR function



can't be

all poles are on $j\omega$ axis

$$T(s) = \frac{N(s)}{D(s)} = \frac{a_m s^m + \dots + a_1 s + a_0}{s^d + b_{d-1} s^{d-1} + \dots + d_0}$$

a_i, b_j real

$s = \sigma + j\omega$

e^{st}

$$s^k \Big|_{s=j\omega} = (j\omega)^k = j^k \omega^k = \begin{cases} (-1)^{k/2} \omega^k & k \text{ even} \\ j(-1)^{\frac{k-1}{2}} \omega^k & k \text{ odd} \end{cases}$$

if $s = j\omega$, $e^{j\omega t} = \cos\omega t + j \sin\omega t$

$D(s)D(-s)\Big|_{s \to -s} = D(-s)D(s) = \text{even} \Big|$ in s

$$T(s) = \frac{N(s)D(-s)}{D(s)D(-s)} = Ev\,T(s) + Od\,T(s)$$

$$T(j\omega) = Ev\,T(j\omega) + Od\,T(j\omega) = R(\omega) + jX(\omega)$$