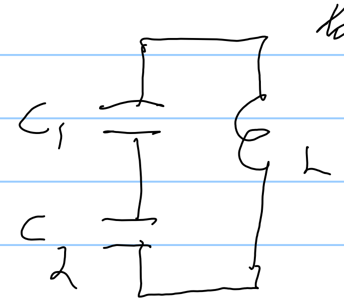
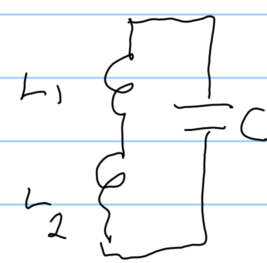
Note Title 11/4/2014

EE303H

11/04-05/14

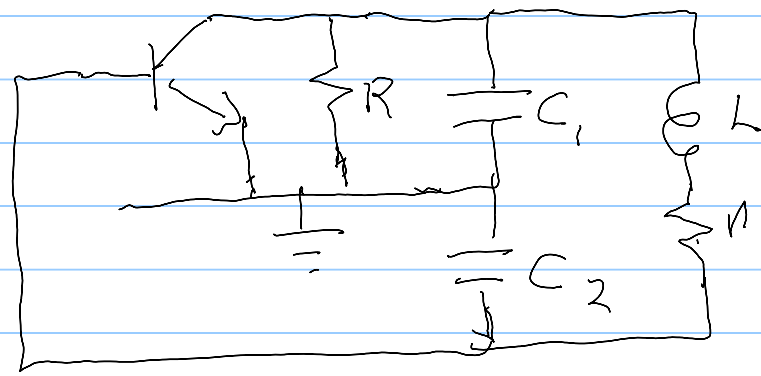
P.1350 Colpitts & Hartley oscillators





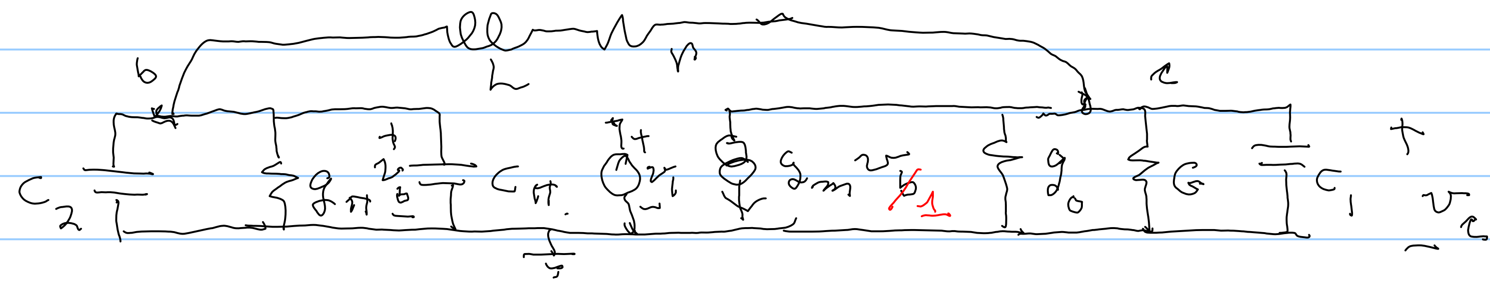
$$\omega_0^2 = \frac{1}{L\left(\frac{C_1 C_2}{C_1 + C_2}\right)}$$

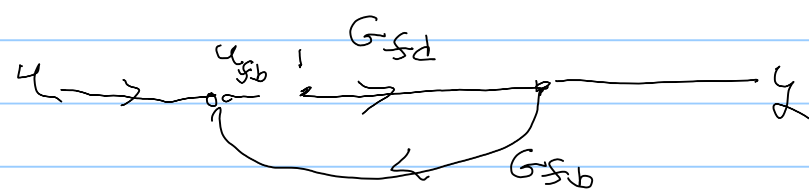
$$\omega_0^2 = \frac{1}{C(L_1 + L_2)}$$



r = loss in inductor

Equivalent circuit:





insert v_1 & find $v_b(s)$ & set $v_1 = v_b(s) = T(s) v_1$

$\Rightarrow 0 = T(s) - 1$ find roots

1) KCL @ c: $0 = -g_m v_1 - (g_0 + G) v_c - sC_1 v_c + \frac{1}{r + sL}(v_b - v_c)$

2) KCL @ b: $0 = \frac{1}{r + sL}(v_c - v_b) - g_\pi v_b - s(C_2 + C_\pi) v_b$

from 2) solve for v_c:

3) $\frac{1}{r + sL} v_c \Rightarrow v_c = \left\{ (r + sL) \left[g_\pi + s(C_2 + C_\pi) \right] + 1 \right\} v_b$

3) $\rightarrow$ 1):

$$0 = -g_m v_1 - \left((g_0 + G) + \frac{1}{r + sL} + sC_1 \right) \left\{ \left(g_\pi + s(C_2 + C_\pi) \right)(r + sL) + 1 \right\} v_b + \frac{1}{r + sL} v_b$$

desire $v_b = v_1$ for oscillation

$$g_m v_1 = -\left[(g_0+G) + \frac{1}{r+sL} + sC_1 \right] \left[\left(g_\pi + s(C_2+C_\pi)\right)(r+sL) + 1 \right] v_b + \frac{1}{r+sL} v_b$$

the $1/(r+sL)$ terms cancel

$$-g_m \frac{v_1}{v_b} = \left[(g_0+G)(r+sL) + 1 + sC_1(r+sL) \right] \left[g_\pi + s(C_2+C_\pi) \right]$$
$$+ \left[(g_0+G) + sC_1 \right] \Rightarrow \text{set } v_1 = v_b$$

cubic in s $\Rightarrow$ set $s = j\omega_0$ get real & imaginary parts to zero. $\Rightarrow$ separate into even & odd polynomials in s

Even: $$0 = g_m + (g_0+G) r \cdot g_\pi + g_\pi + s^2 C_1 L g_\pi + (g_0+G) sL\, s(C_2+C_\pi)$$
$$+ s^2 (C_1 r)(C_2+C_\pi)$$

odd: $$0 = (g_0+G) sL\, g_\pi + sC_1 r g_\pi + (g_0+G) r s(C_2+C_\pi)$$
$$+ s(C_2+C_\pi) + s^3 C_1 L (C_2+C_\pi) + sC_1$$

odd gives; $s = j\omega_0$; from odd ($\div$ out $j\omega_0 \neq 0$)

$$0 = \left[(g_0 + G) L g_\pi + C_1 r g_\pi + (g_0 + G) r (C_2 + C_\pi) + (C_2 + C_\pi) + C_1 \right]$$

$$- \omega_0^2 C_1 L (C_2 + C_\pi)$$

$$\Rightarrow \omega_0^2 = \frac{(C_2 + C_\pi + C_1)}{L C_1 (C_2 + C_\pi)} + \frac{\left[(g_0 + G) L g_\pi + r \{ C_1 g_\pi + (g_0 + G)(C_2 + C_\pi) \right]}{L C_1 (C_2 + C_\pi)}$$

$$= \frac{1}{L C_{eq.}} + \Delta\omega_0^2 \quad ; \quad C_{eq} = \frac{C_1 C_{2\,equiv}}{C_1 + C_{2\,equiv}} = \text{series } C_1 \;\&\; (C_2 + C_\pi)$$

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got before $= \dfrac{v_2}{v_1} = \dfrac{-g_m}{(g_\pi + s C_2) + (G + s C_1)\left[1 + (sL + r)(g_\pi + s C_2) \right]}$

set $C_2 \to \hat{C}_2$, $G \to \hat{G}$

$$-g_m = (g_\pi + \alpha\hat{C}_2) + (\hat{G} + \alpha C_1)\left[1 + (\alpha L + r)(g_\pi + \alpha\hat{C}_2)\right]$$

here $\hat{G} = g_0 + G \quad \hat{C}_2 = C_2 + C_\pi$

$$-g_m = (g_\pi + \alpha\hat{C}_2) + (G + \alpha C_1) + (\hat{G} + \alpha C_1)(g_\pi \alpha L + \alpha^2 L\hat{C}_2 + r g_\pi + \alpha r\hat{C}_2)$$

$$0 = g_m + g_\pi + \alpha\hat{C}_2 + \hat{G} + \alpha C_1 + G g_\pi \alpha L + \hat{G}\alpha^2 L\hat{C}_2 + \hat{G} r g_\pi + G\alpha r\hat{C}_2 + \alpha C_1 g_\pi \alpha L + \alpha^3 L C_1\hat{C}_2 + \alpha C_1 r g_\pi + \alpha C_1 \cdot \alpha r\hat{C}_2$$

$$0 = \alpha^3 L C_1\hat{C}_2 + \left[\hat{G}L\hat{C}_2 + C_1 g_\pi L + C_1 r\hat{C}_2\right]\alpha^2 + \left[\hat{C}_2 + C_1 + \hat{G} g_\pi L + \hat{G} r\hat{C}_2 + C_1 r g_\pi\right]\alpha + (g_m + g_\pi) + \hat{G} + \hat{G} r g_\pi$$

4) $\mathcal{E}_v$: $\left[\hat{G} L \hat{C}_2 + C_1 L g_\pi + C_1 \hat{C}_2 r\right] \alpha^2 + \left[(g_m + g_\pi) + \hat{G} + \hat{G} r g_\pi\right]$

$= 0$

5) Δd: $L C_1 \hat{C}_2 \alpha^3 + \left[C_1 + \hat{C}_2 + \hat{G} g_\pi L + \hat{G} r \hat{C}_2 + C_1 r g_\pi\right] \alpha$

$= 0$ $\Rightarrow$ $\omega_0^2 = \dfrac{1}{L C_1 \hat{C}_2} \left[C_1 + \hat{C}_2 + g_\pi G L + \hat{G} r \hat{C}_2 + C_1 r g_\pi\right]$

$$= \frac{1}{L \dfrac{C_1 \hat{C}_2}{C_1 + \hat{C}_2}} + \underbrace{\frac{g_\pi \hat{G} L + \hat{G} r \hat{C}_2 + C_1 r g_\pi}{L C_1 \hat{C}_2}}_{\Downarrow}$$

$\dfrac{g_\pi \hat{G}}{C_1 \hat{C}_2}$ if $r = 0$

substitute into 4) & solve for g_m $\Rightarrow$ $\dfrac{\hat{C}_2}{C_1} \hat{G}$ if $r \to 0$

$$g_m \simeq -g_\pi - \hat{G}(1 + r g_\pi) + \underline{\hat{G} L \hat{C}_2 + C_1 L g_\pi + C_1 \hat{C}_2 r} \times \omega_0^2$$

set this by bias current $g_m = \dfrac{I_a}{V_T}$