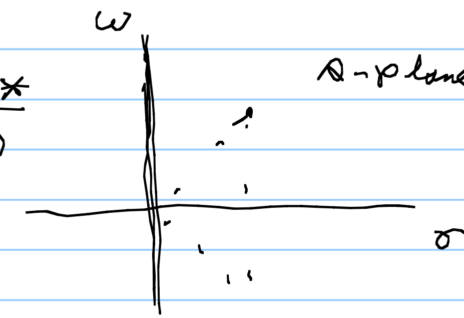
Note Title 10/6/2014

EE303H

10/09/14

all pass $\Rightarrow$ $T(s) = \dfrac{N(s)}{D(s)} \overset{AP}{=} \dfrac{D(-s)}{D(s)} = \dfrac{D_*}{D}$



$s = \sigma + j\omega$

$$\left|T(s)\right|^2\Big|_{s=j\omega} = \left|\frac{N(j\omega)}{D(j\omega)}\right|^2 = \frac{|D(-j\omega)|^2}{|D(j\omega)|^2}$$

$$D(j\omega) = |D(j\omega)|\, e^{j\angle D(j\omega)}$$

if $D(s)$ is polynomial with real coefficients

$$D(j\omega)^* =$$

$* =$ complex conjugate

$j = \sqrt{-1} \Rightarrow -j$

$$D(s) = s^n + d_{n-1}s^{n-1} + d_{n-2}s^{n-2} + \dots d_1 s + d_0$$

$$D(j\omega)^* = (j\omega)^{n*} + d_{n-1}^* (j\omega)^{n-1*} + \ldots d_1^*(j\omega)^* + d_0^*$$

$$= (-j\omega)^n + d_{n-1}(-j\omega)^{n-1} + \ldots d_1(-j\omega) + d_0$$

$$D(j\omega)^* = D(-j\omega)$$

$$|D(j\omega)|^2 = D(j\omega)D^*(j\omega) = D(j\omega)D(-j\omega)$$

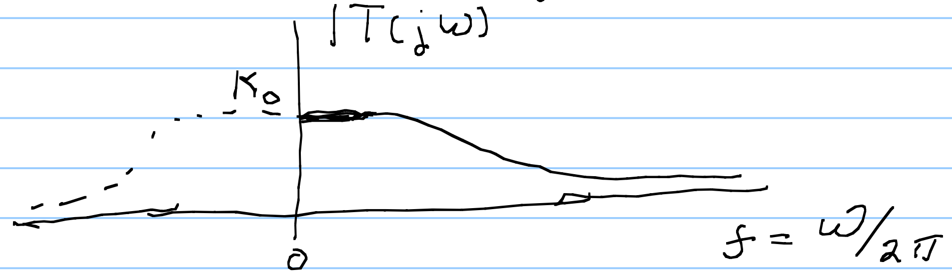
$$|T(j\omega)|^2 = \frac{D(-j\omega)D^*(-j\omega)}{D(j\omega)D^*(+j\omega)} = \frac{D(-j\omega)D(j\omega)}{D(j\omega)D(-j\omega)} = 1$$

$$\angle T(j\omega) = \angle N(j\omega) - \angle D(j\omega) = \angle D(-j\omega) - \angle D(j\omega)$$

$$= -2\angle D(j\omega)$$

$\Rightarrow$ all poles are used for shifting phase

How to specify $T(s)$ for low pass filters



$$T(s) = \frac{K_0 \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

Maximally flat $T(s)$, low pass

$$T(s) = \frac{K_0 d_0}{s^n + d_{n-1} s^{n-1} + \dots + d_1 s + d_0} = \frac{K_0}{\frac{s^n}{d_0} + \frac{d_{n-1} s^{n-1}}{d_0} + \dots \frac{d_1}{d_0} s + 1}$$

let $\hat{s}^n = s^n / d_0$ $\quad = \hat{T}(\hat{s}) = \dfrac{K_0}{\hat{s}^n + \hat{d}_{n-1} \hat{s}^{n-1} + \dots \hat{d}_1 \hat{s} + 1}$

work with $T(s) = \dfrac{K_0}{s^n + d_{n-1}s^{n-1} + \dots + d_1 s + 1}$

desire $\dfrac{d^k}{d\omega}|T(j\omega)| = 0$ for as many k's as possible, $k = 1, 2 \dots$
for maximally flat

$$|T(j\omega)|^2 = \frac{K_0^2}{D(j\omega)D(-j\omega)} \leftarrow \text{an even function of } \omega$$

$$\frac{d\frac{1}{|T(j\omega)|^2}}{d\omega} = -\frac{d|T(j\omega)|^2/d\omega}{(|T(j\omega)|^2)^2} \Leftarrow \neq 0 \qquad \frac{d\,1/f(x)}{dx} = -\frac{df(x)/dx}{f^2(x)}$$

$\therefore$ set as many derivatives of $1/|T(j\omega)|^2\big|_{\omega=0}$ to 0

$\therefore$ derive a Taylor series for $D(j\omega)D(-\omega)$ an even polynomial the Taylor series for a polynomial in x is the polynomial in x

$$K_0^2 \left| \frac{1}{T(j\omega)} \right|^2 = (j\omega)^n(-j\omega)^n + \dots + 1$$

$$= 1 + a_1\omega^2 + a_2\omega^4 + \dots + (-1)^n (j)^{2n} \cdot \omega^{2n}$$

$$= 1 + 0 + \dots 0 + \omega^{2n}$$

$$D(j\omega)(D(-j\omega) = 1 + \omega^{2n} = P(\omega^2)$$ here $s = j\omega \Rightarrow \omega = s/j$

$$D(s)D(-s) = 1 + (s/j)^{2n} = Q(s)$$

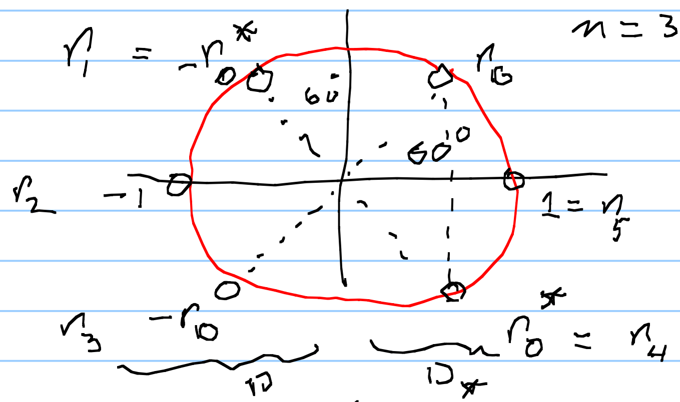
$$= 1 + (-1)^n s^{2n}$$ factor this

$\therefore$ want roots of $(-1)^n s^{2n} + 1 = 0 \Rightarrow (-1)^n s^{2n} = -1$

$$s^{2n} = (-1)^{n+1}$$

$n = \text{odd}:$ desire $2n$ roots of $+1 = e^{j\left(\frac{0 + 2\pi + 2\pi k}{2n}\right)}$ $\quad k = 0, \pm 1, \pm 2$

$n = 3$



$-\alpha^6 + 1 = 0 \implies \alpha^6 = +1$

$k = 0;\quad r_0 = e^{j\pi/3} = e^{j\cdot(4\pi/2n)}$

$k = 1\quad r_1 = e^{j(2\pi/3)}$

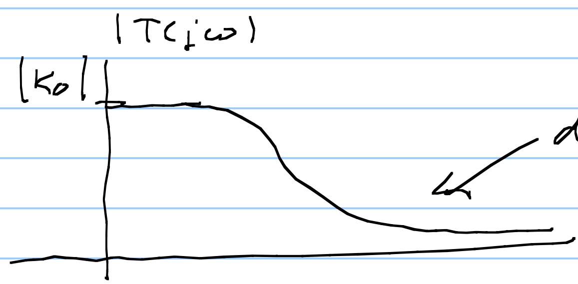
$Q(\alpha) = -\alpha^6 + 1 = -(\alpha - r_0)(\alpha - r_1)(\alpha - r_2)(\alpha - r_3)(\alpha - r_4)(\alpha - r_5)$

$D(\alpha) = (\alpha - r_1)(\alpha - r_2)(\alpha - r_3) = \left(\alpha + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right)(\alpha + 1)\left(\alpha + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$

$= (\alpha + 1)(\alpha^2 + \alpha + 1)$

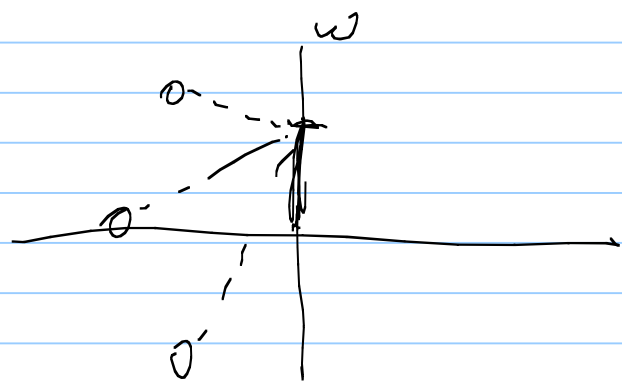
$$T(s) = \frac{K_0}{(s+1)(s^2+s+1)} = \frac{K_0}{s+1} \times \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$\omega_0 = 1, \; Q = 1$

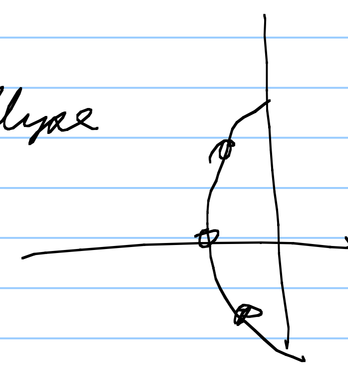


drops as $\frac{1}{\omega^3}$

$f = \omega / 2\pi$



if move to an ellipse



can get equal ripple

Chebyshev

$\cos(n \arccos \omega)$