Note Title 9/28/2014

EE 303H
09/30/14

Differential pairs

MOS p. 595 ⇒ 637 fig. 8.32

i_D's p. 596, eqs. (8.23 & 24)

curves p. 598, fig. 8.7

BJT p. 615

i_E's, p. 616, eqs. (8.73 & 74)

curves, p. 617 ⇒ p. 658, fig. 8.43

The 741 op-amp, p. 1003, fig. 12.13

$$v_{id} = v_{GS_1} - v_{GS_2} = \sqrt{\frac{i_1}{k}} - \sqrt{\frac{i_2}{k}} \qquad i_0 = i_1 - i_2 \;;\; \underline{\underline{i_1 + i_2 = I_T}}$$

$$i_2 = i_1 - i_0 = I_T - i_1 \Rightarrow i_0 = 2i_1 - I_T$$

$$v_{id}^2 = \frac{i_1}{k} + \frac{i_2}{k} - \frac{2\sqrt{i_1 i_2}}{k}$$

$$= \frac{i_1}{k} - \frac{(i_0 - i_1)}{k} - \frac{2}{k}\sqrt{i_1^2 - i_1 i_0} = \frac{2i_1 - i_0}{k} - \frac{2}{k}\sqrt{i_1^2 - i_1 i_0} = \frac{I_T}{k} - \frac{2}{k}\sqrt{i_1^2 - i_1 i_0}$$

$$k v_{id}^2 - I_T = -2\sqrt{i_1^2 - i_1 i_0} \Rightarrow 4\left(i_1^2 - i_1 i_0\right) = \left(I_T - k v_{id}\right)^2$$

$$i_1^2 - i_1 i_0 - \frac{1}{4}\left(I_T - k v_{id}^2\right)^2 = 0 \Rightarrow \text{use } i_0 = 2i_1 - I_T$$

$$\Rightarrow i_1^2 - 2i_1^2 + i_1 I_T - \frac{1}{4}\left(I_T - k v_{id}^2\right)^2 \Rightarrow +i_1^2 - I_T i_1 + \frac{1}{4}\left(I_T - k v_{id}^2\right)^2 = 0$$

$$i_1 = \frac{1}{2} I_T \pm \frac{1}{2}\sqrt{I_T^2 - (I_T - k v_{id}^2)^2} = \frac{1}{2} I_T \pm \frac{1}{2} I_T \sqrt{1 - \left(1 - \frac{k v_{id}^2}{I_T}\right)^2}$$

$$i_2 = I_T - i_1 = \frac{1}{2} I_T \mp \frac{1}{2} I_T \sqrt{1 - 1 + 2\frac{k v_{id}^2}{I_T} - \left(\frac{k v_{id}^2}{I_T}\right)^2} = \frac{1}{2} I_T \left(1 \mp \sqrt{\frac{2k}{I_T}}\, v_{id} \left(1 - \frac{k v_{id}^2}{2 I_T}\right)^{\frac{1}{2}}\right)$$

$$\Rightarrow i_1 = \frac{I_T}{2}\left(1 \pm \sqrt{\frac{2k}{I_T}}\, v_{id} \sqrt{1 - \left(\frac{k}{2 I_T} v_{id}^2\right)^2}\right.$$

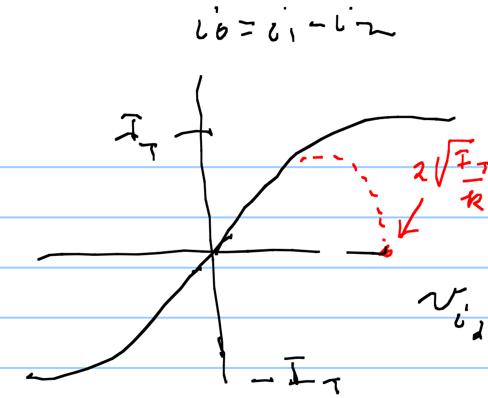
$$i_2 = \frac{I_T}{2}\left(1 \mp \sqrt{\frac{2k}{I_T}} \cdot v_{id} \sqrt{1 - \left(\frac{k}{2 I_T} \cdot v_{id}^2\right)^2}\right.$$

should be (by book)

$$i_{D_1} = \frac{I_T}{2} + \left(\sqrt{k I_T}\right) \frac{v_{id}}{2} \sqrt{1 - \frac{(v_{id}/2)^2}{(I_T/k)}} \qquad \text{requires } (v_{id}/2)^2 < I_T/k$$

$$i_{D_2} = \frac{I_T}{2} - \left(\sqrt{k I_T}\right) \cdot \frac{v_{id}}{2} \sqrt{1 - \frac{(v_{id}/2)^2}{I_T/k}}$$

$$i_o = i_{D_1} - i_{D_2} = 2 \cdot \sqrt{k I_T} \cdot \frac{v_{id}}{2} \sqrt{1 - \frac{k}{I_T}\left(v_{id}/2\right)^2}$$



$$= f(x)\ ; \quad x = \sqrt{\frac{k}{I_T}} \cdot \left(v_{id}/2\right)$$

$$i_o = 2 I_T \sqrt{\frac{k}{I_T}} \cdot \frac{v_{id}}{2} \sqrt{1 - \left(\sqrt{\frac{k}{I_T}} \cdot \frac{v_{id}}{2}\right)^2} = 2 I_T\, x \sqrt{1 - x^2}$$

$i_o = f(x) \Rightarrow$ max: $\frac{d\, f(x)}{dx} = 0$ gives maximum x

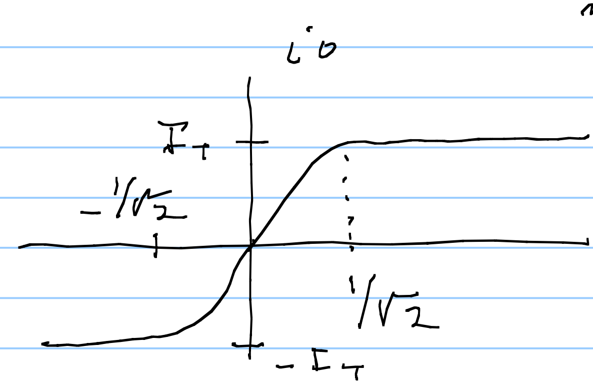
$$\frac{d\, I_T \cdot x \sqrt{1 - x^2}}{dx} = I_T \sqrt{1 - x^2} + I_T\, x \cdot \frac{1}{\sqrt{1 - x^2}} \times (-2x) \times \frac{1}{2}$$

$x^2 \neq 1$

$$= I_T \left\{ \frac{1 - x^2 + x(-x)}{\sqrt{1 - x^2}} \right\} = \frac{I_T\,(1 - 2x^2)}{1 - x^2}$$

max at $x^2 = 1/2 \Rightarrow x_{max} = 1/\sqrt{2} =$

$$i_o = 2I_T \cdot x\sqrt{1-x^2} \Rightarrow i_{o_{max}} = 2I_T \cdot \frac{1}{\sqrt{2}}\sqrt{1-\frac{1}{2}} = \sqrt{2}I_T \cdot \frac{1}{\sqrt{2}} = I_T$$



$$x = \sqrt{\frac{k}{I_T}} \cdot \frac{v_{id}}{2}$$

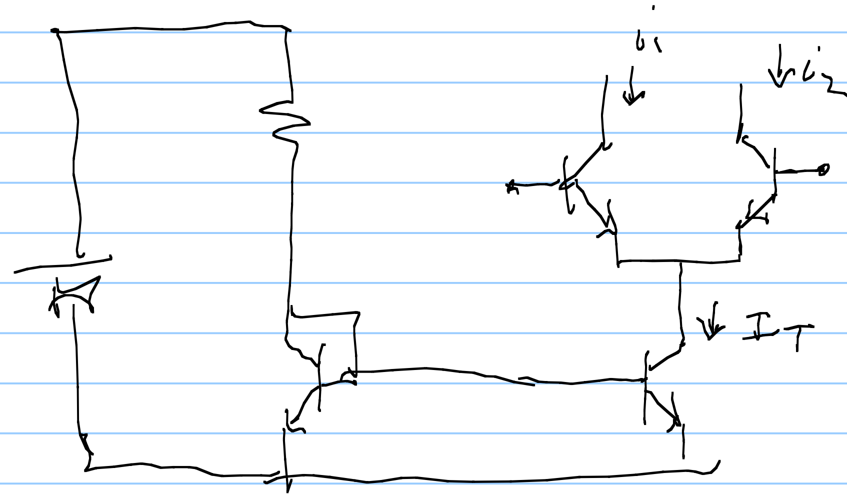
note: $\frac{di_o}{dv_{id}} = \frac{di_o}{dx} \cdot \frac{dx}{dv_i}$; $\frac{di_o}{dx} = 2I_T\left(\sqrt{1-x^2} - x\frac{(-2x)\frac{1}{2}}{(1-x^2)}\right)$

@ $x=0$, $\frac{di_o}{dx} = 2I_T$

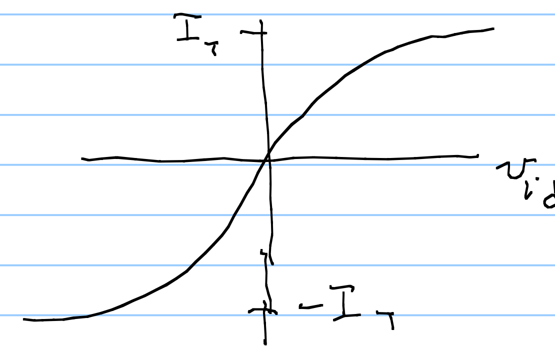
$\Rightarrow g_m = 2I_T \cdot \sqrt{\frac{k}{I_T}} \cdot \frac{1}{2}$

$= \sqrt{kI_T}$

with BJT use similar arguments



$$i_o = i_1 - i_2 = \alpha I_T \tanh\left(\frac{v_{id}}{2V_T}\right)$$



in linear mode, near $v_{id} = 0$

$$i_o = g_m v_{id} \approx 0 + \left.\frac{di_o}{dv_{id}}\right|_{v_{id}=0} (v_{id} - 0) + \ldots$$

$$\frac{d i_o}{d v_{id}} = \alpha I_T \frac{d}{d v_{id}} \tanh\left(\frac{v_{id}}{2V_T}\right) ;$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\frac{d \tanh x}{dx} = \frac{e^{x} + e^{-x}}{e^{x} + e^{-x}} - \frac{(e^{x} - e^{-x})(e^{x} - e^{-x})}{(e^{x} + e^{-x})^2}$$

$$= 1 - \tanh^2 x$$

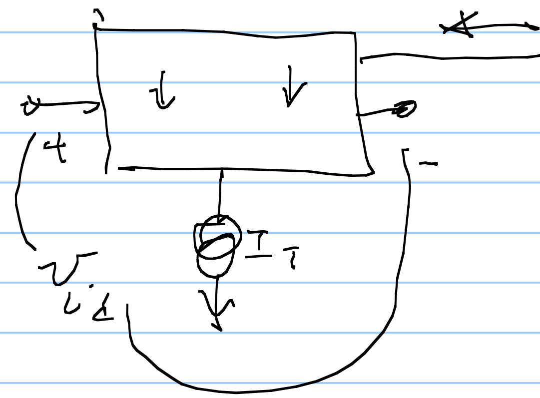
$$= \alpha I_T \cdot \left(1 - \tanh^2\left(\frac{v_{id}}{2V_T}\right)\right) \cdot \frac{1}{2V_T}$$

$$\Rightarrow i_o = 0 + I_T \left(1 - \tanh^2\left(\frac{v_{id}}{2V_T}\right)\right) \frac{1}{2V_T} \Bigg|_{v_{id}=0} (v_{id} - 0) + \dots$$

$$= 1 \cdot \frac{1}{2V_T} \cdot \alpha I_T \cdot v_{id}$$

$$\Rightarrow g_m = \alpha \frac{I_T}{2V_T}$$

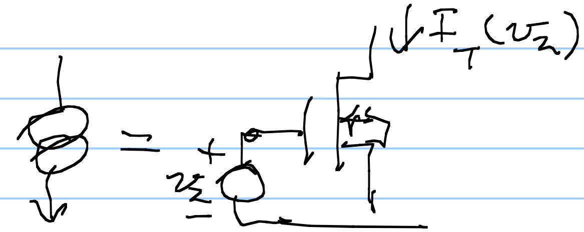
also holds for MOS @ subthreshold

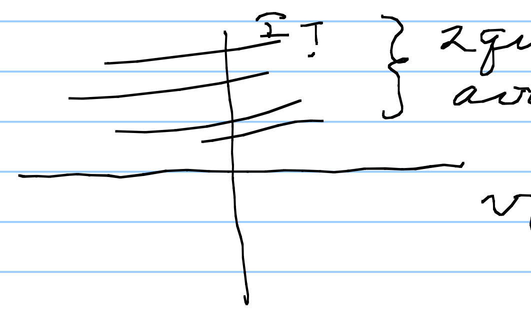


$i_o = I_T \tanh\left(\frac{v_{id}}{2V_T}\right)$

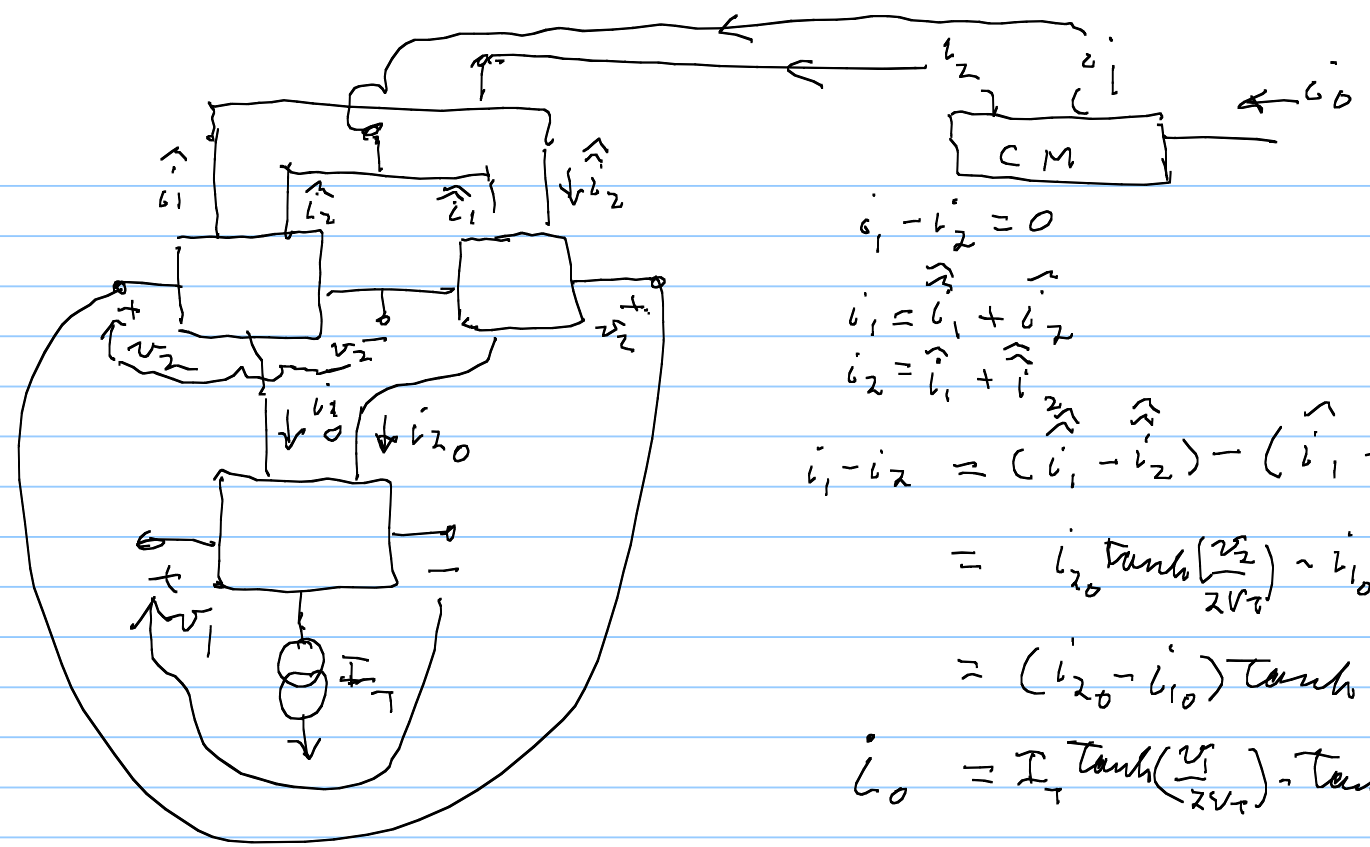
if I_T varies with another voltage then this can become a multiplier

requires $I_T > 0$





} 2 quadrants available



$$i_1 - i_2 = 0$$

$$i_1 = \hat{\hat{i}}_1 + \hat{i}_2$$

$$i_2 = \hat{i}_1 + \hat{\hat{i}}_2$$

$$i_1 - i_2 = (\hat{\hat{i}}_1 - \hat{\hat{i}}_2) - (\hat{i}_1 - \hat{i}_2)$$

$$= i_{2_0} \tanh\left(\frac{v_2}{2V_T}\right) - i_{1_0} \tanh\left(\frac{v_2}{2V_T}\right)$$

$$= (i_{2_0} - i_{1_0}) \tanh\left(\frac{v_2}{2V_T}\right)$$

$$i_o = I_T \tanh\left(\frac{v_1}{2V_T}\right) \cdot \tanh\left(\frac{v_2}{2V_T}\right)$$

for v_1 & v_2 small $\quad i_o = \frac{I_T}{2V_T} \cdot \frac{1}{2V_T} \cdot v_1 \times v_2$