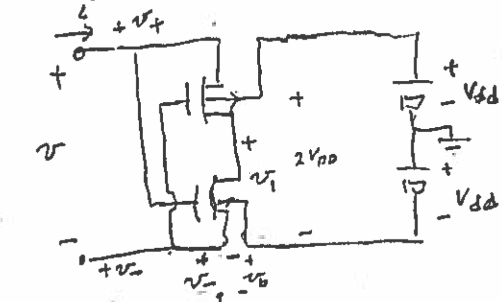
#1 a)



if $v<0$ both off $\qquad v_1 \equiv v/2,\ v_{BS}$

if $0<v<V_{th}$ both off

if $V_{th}<v$ need $v_1 \Rightarrow$ by symmetry $v_1 = v/2$

$\Rightarrow v_{gs} = v,\ v_{GS}-V_{th} = v-V_{th}$ vs $v/2 = v_{DS}$

$\therefore$ saturation for $v-V_{th} \le v/2 \Rightarrow v/2 \le V_{th} \Rightarrow v \le 2V_{th}$

$\Rightarrow$ saturation for $V_{th} \le v \le 2V_{th}$

$\Rightarrow$ ohmic for $2V_{th} \le v$ -

$$\therefore\ i = \begin{cases} 0 & v \le V_{th} \\ k[2(v-V_{th})v/2 - (v/2)^2](1+\lambda v/2) & 2V_{th} \le v \\ k\cdot(v-V_{th})^2(1+\lambda v/2) & V_{th} \le v \le 2V_{th} \end{cases}$$

assumes V_{dd} large enough $= 5V$

To check substrates reversed bias

$0 = -v_- + v_{GS_p} + v_{SB_p} + V_{dd}$ & $0 = -v_+ + v_{GS_n} + v_{SB_n} - V_{dd}$

$\Rightarrow v_- = -v_{SG_p} - v_{BS_p} + V_{dd} \qquad v_+ = v_{GS_n} + v_{SB_n} - V_{dd}$

$v_+ - v_- = v = v + v_{SB_n} - V_{DD} - [-v - v_{BS_p} + V_{dd}] =$

$= 2v + (v_{SB_n} + v_{BS_p}) - 2V_{dd}$

$\Rightarrow 2V_{DD} = v + (v_{SB_n} + v_{BS_p}) = v + 2v_{SB_n}$ by symmetry

$\Rightarrow v_{SB_n} = V_{dd} - v/2 \ge 0$ if $v < 2V_{dd} \Rightarrow$ substrate diodes off

c) assume M_n values; from data $KP = 5.048\times10^{-5}$, $VTO = 0.8582$, $\lambda = 0.0184$

Check @ $v = 4V > 2V_{th} = 2\times0.8582 = 1.716 \Rightarrow i = 50\mu A$ from Spice run

from formula $i = \frac{5.048\times10^{-5}}{2}\times\frac{10^{-5}}{10^{-5}}[2(4-0.8582)\frac{4}{2} - (\frac{4}{2})^2](1+0.0184\times\frac{4}{2}) = 2.524\times10^{-5}((4\times3.1418)-4)(1.036$

$= 2.524\times10^{-5}\times8.882 = 224.2\times10^{-6}$

If use M_p, $KP = 1.908\times10^{-5}$, $VTO = -0.889271$, $\lambda = 0.05012$

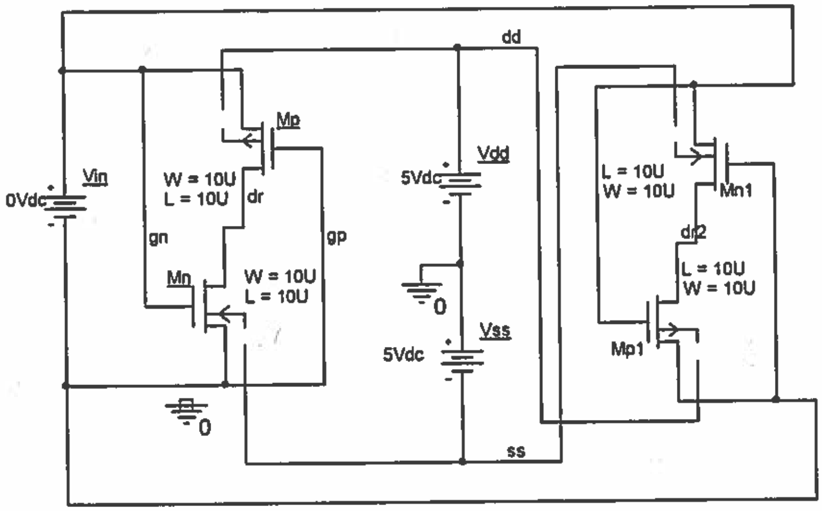
$i = \frac{1.908\times10^{-5}}{2}[2(4-0.889)\frac{4}{2} - (\frac{4}{2})^2](1+0.05012\times2) = 0.9625\times10^{-5}[4(2.111)](1.1024)] = 88.67\times10^{-6}$

which is closer to $50\mu A$ of Spice; error probably due to $V_{th} \ne VTO$ due to V_{SB} (but no γ is given for 4007)

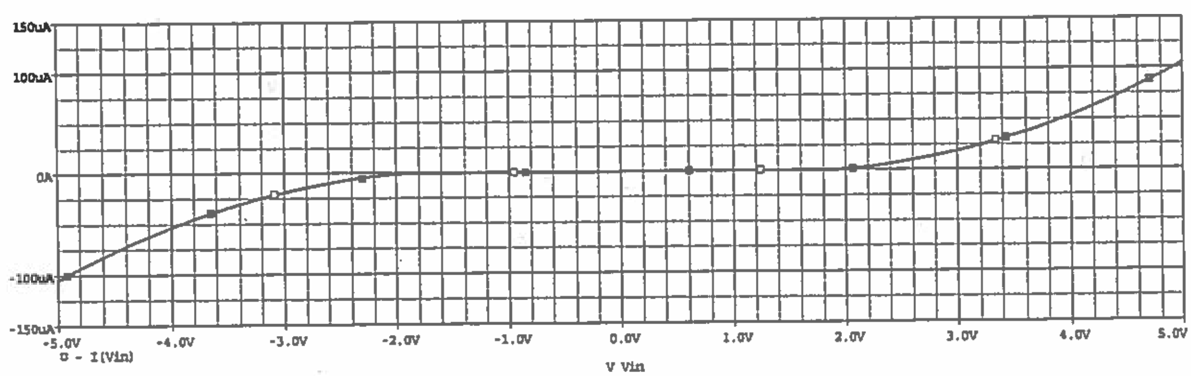
d) If the ground is attached to the bottom of M_n the source and drain of M_n are reversed when $V_{in}<0$ over when $V_{in}>0$ so the bulk-source diode of M_n becomes forward biased.

File g:/coursesF14/303H/final_curve.doc

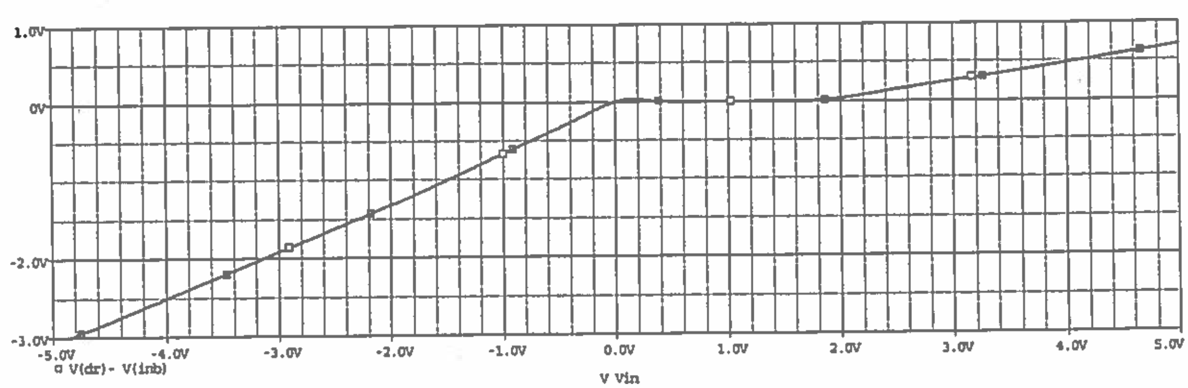
Circuit:



PSpice run for 303H final; above circuit: I vs v:



Vds for Mn:



#2 a) This is for the supply being one voltage source V_{dd} (wrt ground) of large enough magnitude so both transistors are in saturation

b) For V_M including $\lambda \neq 0$, set $I_{D_n} = I_{S_p}$ in saturation states

$$\beta_n (V_M - V_{tn})^2 (1+\lambda V_M) = \beta_p (V_{dd} - V_M - |V_{tp}|)^2 (1+\lambda[V_{dd}-V_M]) \Rightarrow \text{cubic in } V_m$$

$\Rightarrow$ can replace β_n by $\beta_n(1+\lambda V_M)$ & β_p by $(1+\lambda[V_{dd}-V_M])$

$$\Rightarrow \sqrt{\beta_n/\beta_p} \Rightarrow \sqrt{\frac{\beta_n(1+\lambda V_M)}{\beta_p(1+\lambda[V_{dd}-V_M])}} \approx \sqrt{\frac{\beta_n}{\beta_p}} \cdot \sqrt{(1+\lambda V_M)(1-\lambda[V_{dd}-V_M])}$$

$$\approx \sqrt{\frac{\beta_n}{\beta_p}} \cdot \sqrt{1+\lambda[V_M - V_{dd} + V_M]} - (\text{ignore } \lambda^2 \text{ term})$$

$$\approx \sqrt{\frac{\beta_n}{\beta_p}} \cdot (1 - \tfrac{1}{2}\lambda V_{dd} + 2\lambda V_m) \approx \sqrt{\frac{\beta_n}{\beta_p}}(1+\lambda \tfrac{V_{dd}}{2}) \text{ as } V_m \approx V_m|_{\lambda=0} \approx V_{dd}/2$$

$\Rightarrow$ improved V_M

$$V_M = \frac{V_{dd} - |V_{tp}| + V_{tn} \cdot \sqrt{\frac{\beta_n}{\beta_p}} \cdot (1+\frac{1}{2}\lambda V_{dd})}{1 + \sqrt{\frac{\beta_n}{\beta_p}} \cdot (1+\frac{1}{2}\lambda V_{dd})}$$

c) $\beta_n = \frac{KP}{2}\frac{W}{L} = \frac{20.54\times10^{-6}}{2} \times \frac{144\times10^{-6}}{8\times10^{-6}} =$; $\beta_p = \frac{KP}{2}\frac{W}{L} = \frac{10.32\times10^{-6}}{2} \times \frac{328\times10^{-6}}{8\times10^{-6}}$

$\Rightarrow \frac{\beta_n}{\beta_p} = \frac{20.54}{10.32} \times \frac{144}{328} = 0.8738$; $\sqrt{\beta_n/\beta_p} = 0.9348$

$V_{t_n} = VTO = 1.3$, $|V_{t_p}| = |VTO_p| = 1.5$; $\lambda_n = 15\times10^{-3} = \lambda_p = \lambda$

$$V_m|_{\lambda=0} = \frac{6-1.5+1.3\times0.9348}{1+0.9348} = \frac{5.7152}{1.9348} = 2.954$$

$$V_m|_{\lambda\neq0} = \frac{6-1.5+1.3\times0.9348\times(1+0.5\times15\times10^{-3}\times6)}{1+0.9348\times(1+0.5\times15\times10^{-3}\times6)} = \frac{4.5+1.2152\times(1.0075)}{1+0.9348\times(1.0075)} \approx \frac{5.724}{1.942} = 2.947$$

$\therefore$ the Early effect only slightly changes V_m (use of $V_m|_{\lambda=0} \approx V_{dd}$ above gives $V_m = 2.989$)

#3. For the connection $I = I_s e^{0}(1 - e^{-V/V_T}) = I_s(1 - e^{-V/V_T})$

a) $g = \frac{dI}{dV} = I_s(-(-\frac{1}{V_T})e^{-V/V_T}) = \frac{I_s}{V_T} e^{-V/V_T}$

when biased @ $V = V_b \Rightarrow \quad g = \frac{I_s}{V_T} e^{-V_b/V_T}$

b) @ $V_b = 0.6$, $I_s = 20 \, 10^{-18}$, $V_T = 0.026 \Rightarrow V_b/V_T = \frac{6 \times 10^{-1}}{0.26 \times 10^{-1}} = 23.077$

$e^{-V_b/V_T} = 9.502 \times 10^{-11}$, $I_s/V_T = \frac{200 \times 10^{-19}}{26 \times 10^{-3}} = 7.692 \times 10^{-16}$

$\therefore g = 7.692 \times 9.502 \times \times 10^{-16} \times 10^{-11} = 7.309 \times 10^{-26}$

$\Rightarrow r = \frac{1}{g} = \frac{10^{26}}{7.309} = \frac{10}{7.308} \times 10^{25} = 1.368 \times 10^{25}$ $\Rightarrow$ very large!

c) If $V < 0$ the substrate diode is forward biased in which case large currents flow $\Rightarrow$ small r