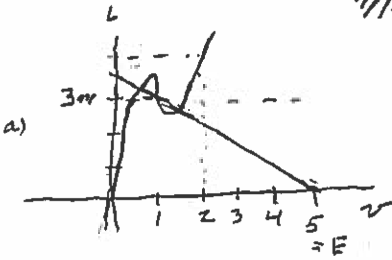
Midterm Solutions F 2014

RWN
10/30–11/04/14
p. 1 of 2

#1. a)



b) $\text{slope} = -\frac{1}{R} = \frac{-3\times10^{-3}}{5-1} = -\frac{3}{4}\times10^{-3} \Rightarrow R = \frac{4}{3}\times10^{3} = 1.33\ldots\,k\Omega$

c) $I_{bias} = i\big|_{v=1} = [3 + 4\times 0]\times10^{-3} = 3\,mA$

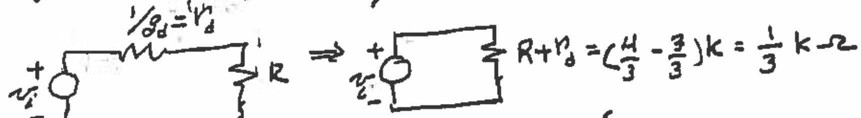
d) $V_{out} = R I_{bias} = \frac{4}{3}\times10^{3}\times 3\times10^{-3} = 4\,v \;\equiv E - V_{bias} = 5-1$

e) $g_d = \frac{\partial i}{\partial v}\Big|_{v=V_{bias}} = 4\times10^{-3}\left[(v-1)(v-\tfrac{3}{2}) + (v-\tfrac{1}{2})(v-\tfrac{3}{2}) + (v-\tfrac{1}{2})(v-1)\right]\Big|_{v=1}$

$= 4\times10^{-3}\left[(1-\tfrac{1}{2})(1-\tfrac{3}{2})\right]\Big|_{v=1} = 4\times10^{-3}\left[\tfrac{1}{2}(-\tfrac{1}{2})\right]$

$= -1\times10^{-3}\,℧$

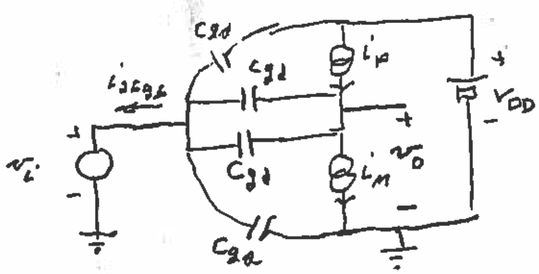
f) The small signal equivalent circuit is



$i = v_i/[R + r_d] = \frac{10^{-3}\cos(2\pi t)}{\frac{1}{3}\times10^{3}} = 3\times10^{-6}\cos 2\pi t$

#2. see attached

#3.



at $v_i = V_{DD}/2$ both M_n & M_p are in saturation
as $|V_{GS}| - |V_{TO}| < \frac{V_{DD}}{2} = |V_{DS}|$

then

$i_p = k(V_{DD} - v_i - |V_{TO}|)^2$

$i_n = k(v_i - V_{TO})^2$

$k = \frac{k'p}{2}\frac{W}{L}$

assume $V_{TO} > 0$

assume $v_i = (1.001)V_{DD}/2$ is near enough to $V_{DD}/2$ to keep M_n & M_p in saturation

$i_p - i_n = i_{2C_{gd}} = 2C_{gd}\frac{d(v_o - v_i)}{dt} = k\left[(V_{DD}-V_{TO})^2 - 2(V_{DD}-V_{TO})v_i + \cancel{v_i^2} - \cancel{v_i^2} + 2V_{TO}v_i + V_{TO}^2\right]$

$= k\left[(V_{DD}^2 - 2V_{TO}V_{DD}) - 2(V_{DD} - 2V_{TO})v_i\right]$

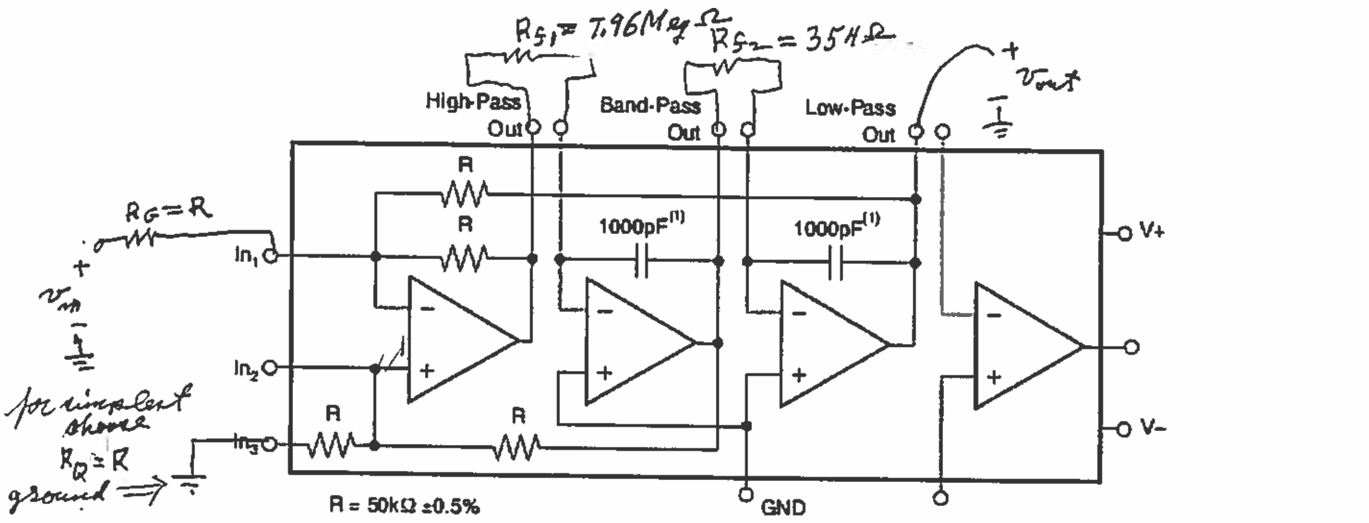
$= k(V_{DD} - 2V_{TO})\left[V_{DD} - 2v_i\right]$

$\therefore 2C_{gd}\frac{dv_o}{dt} = 2C_{gd}\frac{dv_i}{dt} - 2k(V_{DD}-2V_{TO})v_i + k(V_{DD}-2V_{TO})V_{DD}, \quad t > 0$

for $t > 0$, $v_i = 1.001\,V_{DD}/2$, $dv_i/dt = 0$

$\Rightarrow \frac{dv_o}{dt} = -\frac{k}{C_{gd}}(V_{DD}-2V_{TO})(1.001)\frac{V_{DD}}{2} + \frac{k}{C_{gd}}(V_{DD}-2V_{TO})\frac{V_{DD}}{2} = -(0.001)\frac{k}{C_{gd}}(V_{DD}-2V_{TO})\frac{V_{DD}}{2}$

$= -0.001\frac{k}{C_{gd}}(V_{DD}-2V_{TO})\cdot\frac{V_{DD}}{2}, \quad t > 0, \quad v_o(0_+) = V_{DD}/2$



NOTE: (1) ±0.5%.

#2. for $A_0 = -1 = -\frac{R_1}{R_G} = -\frac{R}{R_G} \Rightarrow R_G = R = 50k\Omega$

for $\omega_m^2 = (6\pi \times 10^3)^2 = \frac{R_2}{R_1 R_{f_1} R_{f_2} C_1 C_2} = \frac{R}{R R_{f_1} R_{f_2} C^2} \Rightarrow R_{f_1} R_{f_2} = \frac{1}{\omega_m^2 \times (10^3 \times 10^{-12})^2}$

$= \frac{10^{18}}{36\pi^2 \times 10^6} = \frac{10^{12}}{36\pi^2} \approx 28.4 \times 10^8$

for $Q = 100 = \left[1 + \frac{R_4}{R_Q}\right]\left[\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_G}}\right]\left[\frac{1}{R^2}\frac{R_{f_1} C_1}{R_{f_2} C_2}\right]^{1/2}$

$= \left[1 + \frac{R}{R_Q}\right]\left[\frac{R}{3} \cdot \frac{1}{R}\right]\left[\frac{R_{f_1}}{R_{f_2}}\right]^{1/2} \Rightarrow \frac{R_{f_1}}{R_{f_2}} = \frac{10^4 \cdot 3^2}{(1 + R/R_Q)^2} = \frac{9 \times 10^4}{(1 + R/R_Q)^2}$

$\therefore R_{f_1} = \frac{10^{12}}{36\pi^2} \cdot \frac{1}{R_{f_2}} \Rightarrow \frac{R_{f_1}}{R_{f_2}} = \frac{10^{12}}{36\pi^2} \cdot \frac{1}{R_{f_2}^2} = \frac{9 \times 10^4}{(1 + R/R_Q)^2}$

$\Rightarrow R_{f_2}^2 = \frac{10^{12}}{36\pi^2} \times \frac{1}{9} \times \frac{1}{10^{+4}} \times (1 + R/R_Q)^2 = 10^{+8} \times \frac{1}{9 \times 36\pi^2} \cdot (1 + R/R_Q)^2$

$\Rightarrow R_{f_2} = \frac{1}{3 \times 6\pi} \times 10^4 (1 + R/R_Q)$

for simplest choose $R_Q = R \Rightarrow R_{f_2} = \frac{1 \times 2}{3 \times 6\pi} \times 10^4 = \frac{1}{9\pi} \times 10^4$; $R_{f_1} = \frac{10^{12}}{36\pi^2} \times \frac{1}{\frac{1}{9\pi} \times 10^4} = \frac{10^8}{4\pi} = \frac{25}{\pi} \times 10^6$

$\approx 0.03537\ldots \times 10^4$ $\approx 7.958 \times 10^6$

$\approx 354\Omega$ ≈ 7.96 Meg Ω