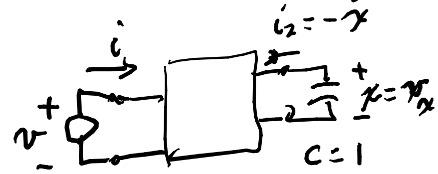
PR scalar: $y(s) = \dfrac{s+1}{s+2} = \dfrac{1}{1+\frac{2}{s}} + \dfrac{1}{s+2} \Rightarrow$ is PR as can make with a passive circuit

$$y(s) = 1 + \underbrace{\frac{-1}{s+2}}_{\text{not PR}}$$

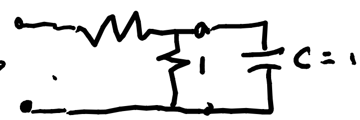
$$\dot{x} = -2x + 1\cdot u \qquad u = v$$
$$i = -1x + 1\cdot u$$



$$\begin{bmatrix} D & C \\ -B & -A \end{bmatrix} \begin{bmatrix} v \\ x \end{bmatrix} = \begin{bmatrix} i \\ -\dot{x} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & +2 \end{bmatrix} \begin{bmatrix} v \\ x \end{bmatrix} = u$$

$$Y = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}, \quad \det Y = 2 - 1 > 0$$

$y(s) = \dfrac{s+1}{s+2} \rightarrow$



Ex: $y(s) = \dfrac{2s+8}{s+2} = 2 + \dfrac{4}{s+2}$

$$\dot{x} = -2x + 1u \qquad Y = \begin{bmatrix} D & C \\ -B & -A \end{bmatrix}$$
$$i = 4x + 2\cdot u$$

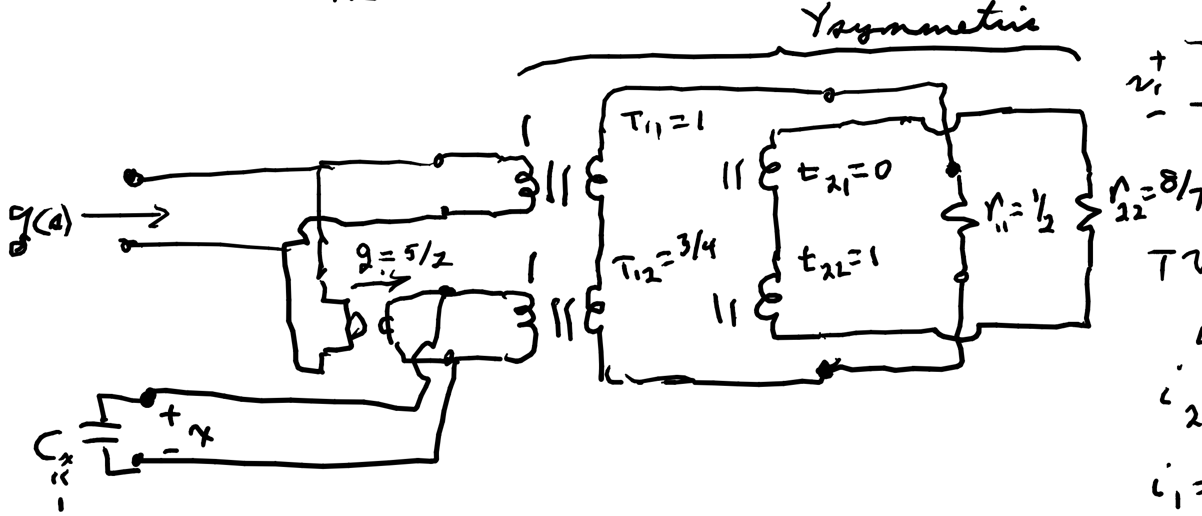
$$Y = \begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 3/2 \\ 3/2 & 2 \end{bmatrix}}_{\frac{Y+Y^T}{2}} + \underbrace{\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}}_{\frac{Y-Y^T}{2}}$$

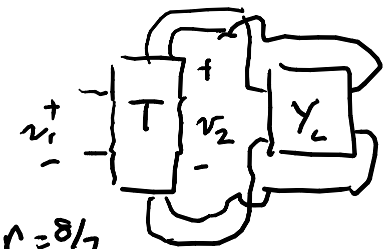
$$\begin{bmatrix} 1 & 0 \\ -\frac{3}{4} & 1 \end{bmatrix} \begin{bmatrix} 2 & 3/2 \\ 3/2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3/2 \\ 0 & 2 - 9/8 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 3/2 \\ 0 & 7/8 \end{bmatrix}$$

$$\det\left(\frac{Y+Y^T}{2}\right) = 4 - 9/4 = \frac{16-9}{4}$$

$$\begin{bmatrix} 2 & 3/2 \\ 0 & 7/8 \end{bmatrix} \begin{bmatrix} 1 & -3/4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 7/8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3/2 \\ 3/2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3/4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 7/8 \end{bmatrix} \begin{bmatrix} 1 & 3/4 \\ 0 & 1 \end{bmatrix} = T^T Y_L T$$





$T v_1 = v_2$

$i_1 = -T^T i_2$

$i_2 = -Y_L v_2$

$i_1 = T^T Y_L T v$

$Y = T^T Y_L T$

Ex: $y(s) = \dfrac{2s+12}{s+2} = 2 + \dfrac{8}{s+2}$

$\dot{x} = -2x + 1u$

$i = 8x + 2 \cdot u$

$$Y = \begin{bmatrix} 2 & 8 \\ -1 & +2 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 7/2 \\ 7/2 & 2 \end{bmatrix}}_{Y_{sym}} + \begin{bmatrix} 0 & 9/2 \\ -9/2 & 0 \end{bmatrix}$$

$\det Y_{sym} = 4 - \dfrac{49}{4} = \dfrac{16-49}{4}$

$= -33/4$

use PR lemma:

$-PA - A^T P = -p(-2) - (-2)p = 2p = \sqrt{2p} \cdot \sqrt{2p}\ ; \ p > 0$

$p = Q \cdot Q^T = \sqrt{2p} \cdot \sqrt{2p}\ ; \quad Q = \sqrt{2p}$

$\hat{x} = Qx \qquad \dot{x} = -2x + 1 \cdot u \Rightarrow Q\dot{x} = Q(-2) \cdot Q^{-1} Q x + QBu$

$\dot{\hat{x}} = (\sqrt{2p}) \cdot (-2) \dfrac{1}{\sqrt{2p}} \cdot \hat{x} + \sqrt{2p} \cdot 1u$

$i = Cx + Du = CQ^{-1}Qx + Du = 8 \dfrac{1}{\sqrt{2p}} \hat{x} + 2u$

$$\hat{Y} = \begin{bmatrix} 2 & 8/\sqrt{2p} \\ -\sqrt{2p} & 2 \end{bmatrix} ; \quad Y_{sym} = \begin{bmatrix} 2 & \dfrac{8}{\sqrt{2p}} - \sqrt{2p} \\ \dfrac{\frac{8}{\sqrt{2p}} - \sqrt{2p}}{2} & 2 \end{bmatrix}$$

$$\det Y_{sym} = 4 - \left(\frac{8-2p}{2\sqrt{2p}}\right)^2 = 4 - \frac{64+32p-4p^2}{8p}$$

$$= -\frac{4p^2+64p-64}{8p}$$

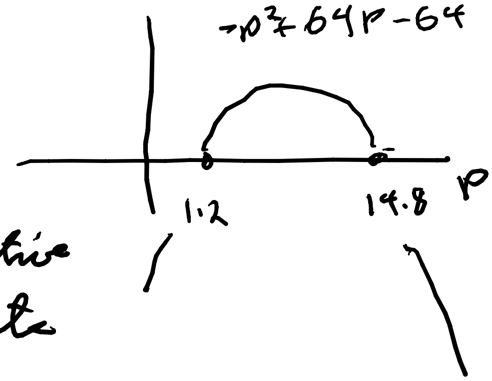
we want this ≥ 0

$$p^2 - 16p + 16 = 0$$

$$p = \frac{16}{2} \pm \frac{1}{2}\sqrt{16^2 - 4\times 16}$$

$$= 8 \pm \frac{8}{2}\sqrt{4-1} = 8 \pm 4\sqrt{3}$$

$$\approx 8 \pm 4\times 1.732 = 8 \pm 6.8$$



∴ choose
$1.2 < p < 14.8$

then Y_{sym} is positive semi definite

$$Y_{sym} = T^T \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \end{bmatrix} T, \quad g_{11} > 0, \; g_{22} > 0$$

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for $v_o/v_i = T(s) \Rightarrow$

$$\dot{x} = Ax + Bv_i$$

$$v_o = Cx + Dv_i$$

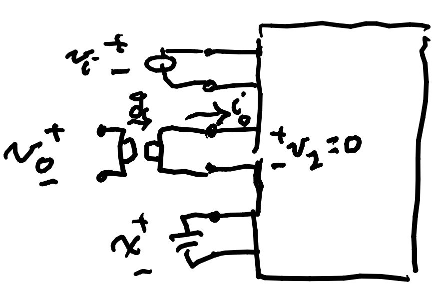
$\Downarrow$

i_o 1st

$$\begin{bmatrix} i_i \\ i_o \\ -\dot{x} \end{bmatrix} = \begin{bmatrix} x & x & x \\ D & \underset{\tilde{x}}{x} & C \\ -B & & -A \end{bmatrix} \begin{bmatrix} v_i \\ v_2 \\ x \end{bmatrix}$$

x = don't care

choose x's to make as skew as possible



$$Y = \begin{bmatrix} 0 & -D^T & +B^T \\ D & 0 & C \\ -B & -C^T & -A \end{bmatrix}$$