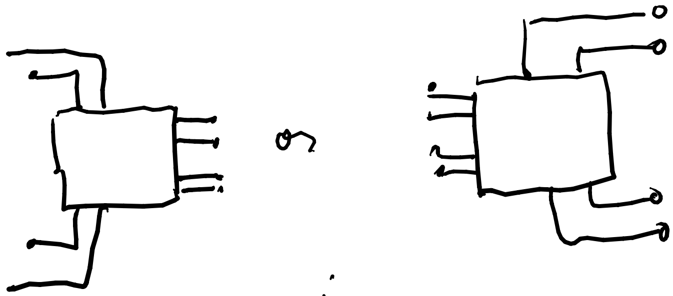
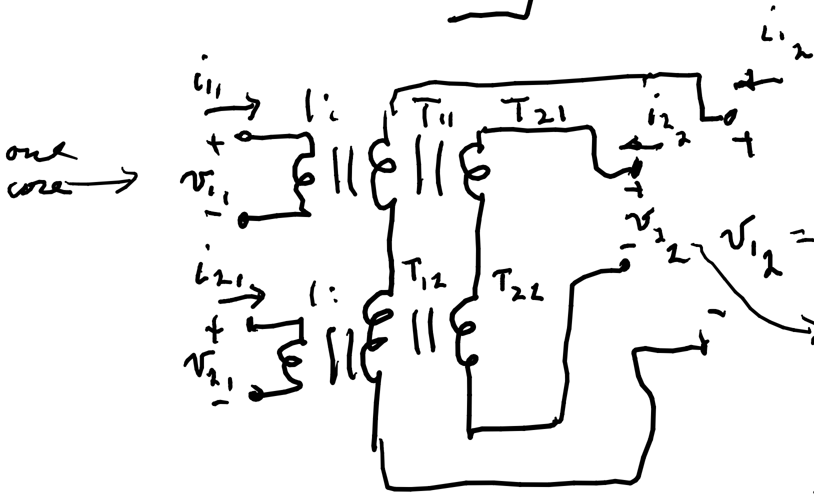
Transformers





$$v_{1_2} = T_{11} v_{1_1} + T_{12} v_{2_1}$$

$$v_{2_2} = T_{21} v_{1_1} + T_{22} v_{2_1}$$

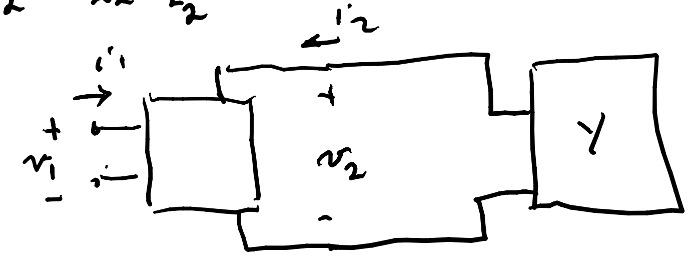
$$v_2 = T v_1 \quad ; \quad v_1 = \begin{bmatrix} v_{1_1} \\ v_{2_1} \end{bmatrix}, \; v_2 = \begin{bmatrix} v_{1_2} \\ v_{2_2} \end{bmatrix}$$

$$i_1 = -T^T i_2$$

$\sum$ amp turns $= 0$

$$1 \cdot i_{1_1} + T_{11} i_{1_2} + T_{21} i_{2_2} = 0$$

$$1 \cdot i_{2_1} + T_{12} i_{1_2} + T_{22} i_{2_2} = 0$$



$i_2 = Y(-v_2)$

(Y is 2×2 for above 4-port (transformer))

$$v_2 = T v_1$$
$$i_1 = -T^T i_2$$

$$v_2 = T v_1 , \quad i_1 = -T^T i_2 = -T^T(-Y v_2) = +T^T Y T v_1$$

∴ input of transformer $Y_T = T^T Y T$ (a congruency transformation)

For state variable eqs.

$$\dot{x} = Ax + Bu \qquad\qquad T(s) = \text{transfer function}$$
$$y = Cx + Du \qquad\qquad y(s) = T(s)\,u(s)$$

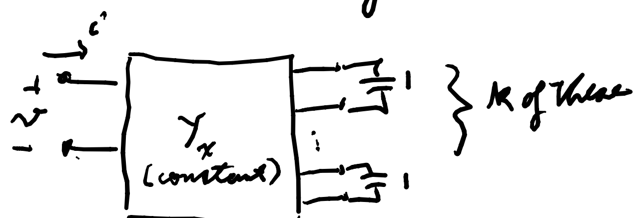
$$T(s) = C(s1 - A)^{-1}B + D$$

$\times\, Q$ on $x \Rightarrow \quad Qx = \hat{x}, \quad \dot{Q} = 0_k$

$$\dot{\hat{x}} = Q\dot{x} = QAQ^{-1}Qx + QBu$$
$$y = CQ^{-1}Qx + Du$$

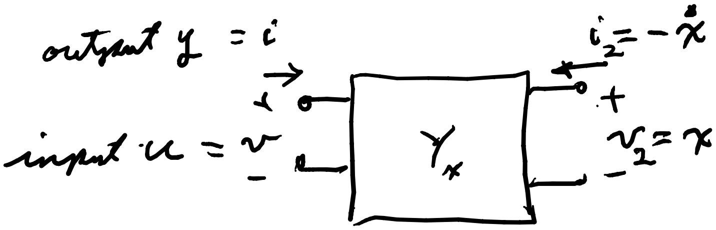
$$\dot{\hat{x}} = QAQ^{-1}\hat{x} + QBu \qquad\qquad y(s) = T(s)\,u(s)$$
$$y = CQ^{-1}\hat{x} + Du \quad \Rightarrow \quad T(s) = CQ^{-1}(s1_k - QAQ^{-1})^{-1}QB + D$$
$$= CQ^{-1}(sQ^{-1}1_k - Q^{-1}\cdot QAQ^{-1})^{-1}B + D$$
$$= C(sQ^{-1}1_kQ - Q^{-1}QAQ^{-1}Q)^{-1}B + D$$
$$= C(s1_k - A)^{-1}B + D$$

$\Rightarrow$ transfer function is unchanged but internal state is modified



let x = voltages on capacitors

then $-\dot{x}$ = current out of capacitors



$$\begin{bmatrix} i \\ i_1 \end{bmatrix} = Y_x \begin{bmatrix} v \\ v_1 \end{bmatrix}$$

$$\begin{bmatrix} i \\ -\dot{x} \end{bmatrix} = \begin{bmatrix} D & C \\ -B & -A \end{bmatrix} \begin{bmatrix} v \\ x \end{bmatrix}$$

$$Y_x = \begin{bmatrix} D & C \\ -B & -A \end{bmatrix} = Y_{x\,\text{symmetric}} + Y_{x\,\text{skew symmetric}}$$

|| transformers & resistors || gyrators

$P\, Y_{x_{sym}} P^T = \text{diagonal}[g_1 \cdots g_{k+1}]$; passive = desire positive

$$= \begin{bmatrix} g_1 & & 0 \\ & \ddots & \\ 0 & & g_{k+1} \end{bmatrix} \Rightarrow Y_{x_{sym}} = P^{-1} \,\text{diag}[g_1 \cdots g_{k+1}]\, P^{-T}$$

$\Rightarrow$ use a transformer of turns ratio matrix $T = P^{-T}$

by the PR lemma there exists a Q to guarantee that $\hat{Y}_{\hat{x}} = \begin{bmatrix} D & CQ^{-1} \\ -QB & -QAQ^{-1} \end{bmatrix}$ is PR (constant matrix) $\Rightarrow$ guarantees only non negative g_i

$$2\hat{Y}_{\hat{x}_{sym}} = \hat{Y}_{\hat{x}} + \hat{Y}_{\hat{x}}^T$$

$$= \begin{bmatrix} D + D^T & CQ^{-1} - B^T Q^T \\ -QB + Q^{-T} C^T & -QAQ^{-1} - Q^{-T} A^T Q^T \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & Q \end{bmatrix} \begin{bmatrix} D + D^T & CQ^{-1}Q^{-T} - B^T \\ -B + Q^{-1}Q^{-T}C^T & -AQ^{-1}Q^{-T} - Q^{-1}Q^{-T}A^T \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & Q^T \end{bmatrix}$$

essentially transformer

The PR lemma is that there is $P = Q^{-1}Q^{-T}$ such that the middle matrix is positive semi definite

$\therefore$ can diagonalize the middle Y by congruency transformations to $1_{k_0} \dotplus 0_{k+1-k_0} = \begin{bmatrix} 1_{k_0} & 0 \\ 0 & 0_{k+1-k_0} \end{bmatrix}$

$\Rightarrow$ can make with transformers & +1 ohm resistors